VOLUME 78 SEPARATE No. 144A 144B

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

AUGUST, 1952

AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

PROGRESS REPORT OF THE ADVISORY BOARD ON THE INVESTIGATION OF SUSPENSION BRIDGES

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

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Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Proceedings—Vol. 78 August, 1952 Separate No. 144A 144B

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

PROGRESS REPORT OF THE ADVISORY BOARD ON THE INVESTIGATION OF SUSPENSION BRIDGES

Preface

The Advisory Board on the Investigation of Suspension Bridges was organized in 1942 by Thomas H. MacDonald, Hon. M., ASCE, Commissioner of Public Roads, at the request of a representative group of engineers concerned with the problems of suspension bridge design. A list of the sponsor organizations and their representatives is given in Sect. 25.

The purpose of the Advisory Board is to coordinate research dealing with suspension bridges with particular regard to their aerodynamic stability. It has acted in an advisory capacity with respect to the following projects and many of the specific investigations made have been initiated at the suggestion of the Board:

1. Cooperation of the Washington Toll Bridge Authority, the University of Washington and the Bureau of Public Roads in a study of the effect of aerodynamic forces on models of suspension bridges and a theoretical and experimental study of means for predicting the aerodynamic behavior of actual suspension bridges.

Numerous progress reports have been submitted to the Board and a final report (3)[1] in five parts is being prepared. Part I "Investigations Prior to October, 1941" and Part II "Mathematical Analyses" have been printed; Part III covering tests on models of the original Tacoma Narrows Bridge, is being printed. Part IV, covering tests on models of the new Tacoma Narrows Bridge, and Part V, covering more general research on suspension bridge aerodynamics are being prepared.

The studies are continuing.

Note.—Written comments are invited for publication; the last discussion should be submitted by February 1, 1953.

[1] Numerals in parentheses, thus: (3), denote corresponding items in the list of references, Appendix I.

2. Cooperation of the American Institute of Steel Construction and the Bureau of Public Roads in theoretical analyses of the action of suspension bridges when subjected to aerodynamic forces.

Twelve reports by the late Friedrich Bleich, M. ASCE, have been submitted to the board and incorporated in two subsequent publications (1) (11) (See item 7).

3. Cooperation of the Golden Gate Bridge and Highway District and the Bureau of Public Roads in making observations of the movements of the Golden Gate Bridge. Progress reports have been submitted to the Board. Brief news items concerning these investigations have been published.

4. A study of the damping characteristics of structural steel members in the laboratory of the Bureau of Public Roads. Reports have been submitted to the Board and the test results have been published (17).

5. Cooperation of the Oregon State Highway Commission and the Bureau of Public Roads in static model tests to verify Friedrich Bleich's Linearized Deflection Theory, particularly adapted for evaluating the effect of diagonal tower stays and mid-span cable ties on the behavior of the bridge under static loading. A report has been printed (1) (See item 7).

6. Publication by the Agricultural and Mechanical College of Texas in cooperation with the Bureau of Public Roads of a reprint of the three original official reports of the failure of the Tacoma Narrows Bridge. In 1941, shortly before the organization of the Advisory Board, there was also published (25) by the Agricultural and Mechanical College of Texas in cooperation with the Bureau of Public Roads, "A History of Suspension Bridges in Bibliographical Form," by A. A. Jakkula.

7. Cooperation of the Oregon State Highway Commission and the Bureau of Public Roads in the preparation of the manuscript (1) for "The Mathematical Theory of Vibration of Suspension Bridges" which has been printed by the Bureau of Public Roads.

The Board has cooperated, through the exchange of information, with the National Physical Laboratory of Great Britain, which has made wind tunnel studies for the proposed suspension bridge over the Severn River near Bristol, England.

Although the investigations have not been completed, it is possible now to incorporate certain findings and tentative recommendations in a preliminary report. In addition to the above listed projects, the Board has drawn upon the reports of other investigations and the published writings of others in this field.

This report was prepared for the Advisory Board on the Investigation of Suspension Bridges by George S. Vincent, M. ASCE, Principal Highway Bridge Engineer, Bureau of Public Roads, and has been reviewed, amended and officially approved by the Board. Because of his close association with the investigations with which the Board has been concerned and his intimate knowledge for the subject, Mr. Vincent was well qualified for this task.

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SYNOPSIS

The aerodynamic forces which act upon a bridge in the wind depend only upon the velocity and direction of the wind and the size, shape, and motion of the bridge. The occurrence of resonance involving these wind forces with the motion of the bridge depends upon the same factors. The amplitude of oscillation which may be built up depends upon the strength of the wind forces (including their variation with amplitude) the energy storage capacity of the structure, the structural damping and the duration of a wind capable of exciting motion.

The wind velocity and direction (including vertical angle) can be determined by extended observations at the site. They can be approximated with reasonable conservatism on the basis of a few local observations and extended study of more general data. The choice of the wind conditions for which a given bridge should be designed may always be largely a matter of judgment.

The size and shape of the bridge are, of course, known. Its motion, consisting essentially of natural modes of vibration, is determined completely by its mass, mass distribution, and elastic properties, from which the motion and energy storage capacity can be computed by reliable methods.

The only unknown element is that factor relating the wind to the bridge section and its motion. This is a highly individual factor which can not at present be generalized but is subject to reliable determination in each case. Properties of the bridge including its elastic forces and its mass and motions (determining its inertial forces) can be computed and reduced to model scale, and wind conditions bracketing all probable conditions at the site can be imposed upon a section model. The motions of such a dynamic section model in the properly scaled wind will duplicate reliably the motions of a convenient unit length of the bridge. The wind forces and the rate at which they can build up energy of oscillation respond to the changing amplitude of the motion, and the rate of energy change can be measured and plotted against amplitude. Thus the section model test measures the one unknown factor which can then be applied by calculation to the variable amplitude of motion along the bridge to predict the full behavior of the structure under the specific wind conditions of the test. These predictions are not precise but are about as accurate as some other features of the design analysis.

As stated, this factor, relating bridge movement to wind conditions, is highly individual; therefore, detailed criteria for the design of favorable bridge sections can not be written until a large mass of individual data has been accumulated. However, it can be stated that:

(1) In general, a truss-stiffened section is more favorable than a girder-stiffened section.
(2) Deck slots and other devices which tend to break up the uniformity of wind action are likely to be favorable.
(3) The use of two planes of lateral system to form a four-sided truss can materially increase the frequency of any torsional motion. Such design strongly inhibits flutter and also raises the critical velocity of a pure torsional motion.

(4) For a given section a high natural frequency of vibration is usually favorable—
   (a) For short to moderate spans a useful increase in frequency, if needed, can be attained by increased truss stiffness. (Although not closely defined, the term "moderate spans" may be regarded as including lengths to about 1800 ft.)
   (b) For long spans it is not economically feasible to obtain any material increase in natural frequency of vertical modes above that inherent in the span and sag of the cable.
   (c) The possibility should be considered that for longer spans in the future, with their unavoidable low natural frequencies, oscillations due to any unfavorable aerodynamic characteristics of the cross section may be more prevalent than in the case of bridges of moderate span.

(5) It appears probable that at most bridge sites the wind is broken up—that is, non-uniform across the site, unsteady and turbulent so that a condition which could cause serious oscillation does not continue long enough to build up an objectionable amplitude. However,
   (a) There are undoubtedly sites where the winds from certain quarters are unusually steady and uniform.
   (b) There are bridge sections on which any wind over a wide range of velocity will continue to build up some mode of oscillation.

(6) An increase in stiffness arising from increased weight increases the energy storage capacity of the structure without increasing the rate at which the wind can contribute energy, and thereby increases the time required to build up an objectionable amplitude. This may have a beneficial effect much greater than is suggested by the percentage increase in weight because of the sharply reduced probability of the wind continuing unchanged for the greater length of time. Increased stiffness may give added structural damping and other favorable results.

Although more specific design criteria than the above cannot be given, it is possible to design a suspension bridge with a high degree of security against aerodynamic forces. This involves the calculation of natural modes of motion of the proposed structure, the performance of dynamic section model tests to determine the factors of behavior, and the application of these factors to the prototype by suitable analysis. This analysis is not more complicated than that commonly applied to the design of the structure for static loading although at present it is less familiar.

If the analysis forecasts an unfavorable aerodynamic behavior it will also suggest the general line of modification which is likely to result in improvement at least expense. The modified design must then be investigated. This procedure is similar to that by which the most economical and satisfactory of several alternate designs is usually established.

Introduction

The Advisory Board recognizes the obligation to make this preliminary report as concise as possible consistent with clear presentation. However, this relatively new subject (aerodynamic vibration of suspension bridges) in the literature of civil engineering must be presented without presuming on the reader's familiarity with the background and fundamentals. The intent is to cover all phases of the problem but not to burden the text with detailed proof or analysis, using instead citations to specific references in Appendix I, elaborating the statements made so that the reader can satisfy himself as to each point in question. The list of references is not complete but has been selected for conciseness.

The nature of wind action and of the behavior of the bridge in response to it are discussed in Part I. It is hoped that this will throw light on the mechanics of the phenomenon, give an appreciation of the relative influence of the many factors involved, give clues to the design features which can be manipulated to improve the stability of a bridge and suggest the type of investigation required to determine its probable behavior.

In Part II are discussed the means by which the aerodynamic characteristics of a design may be improved and a statement is given of tentative criteria for making a bridge safe against objectionable oscillation in the wind. An analytical and test procedure for confirming the adequacy of the design is presented.

Part III reviews the facts which have been established and points out the features which yet remain to be investigated before a fully satisfactory basis can be established for designing suspension bridges against objectionable oscillation under wind action.

I.—THE NATURE OF WIND ACTION AND OF THE BRIDGE RESPONSE

1. Wind Pressure

(A) *Static Forces.*—The principal force exerted on a bridge by a steady wind is a substantially constant pressure acting in the general direction of the wind. This constitutes the conventional wind loading taken into account in bridge design. The intensity of this pressure increases as the square of the velocity and is expressed by the formula (see any reference book on fluid mechanics):

$$p = C \frac{\rho}{2} V^2 \quad \text{(1)}$$

in which p is in pounds per square foot, ρ is the mass density of the air (0.00238 slugs per cubic foot at sea level and 15° C.), V is the velocity in feet per second and C is a constant depending upon the shape and size of the obstruction. The term, $\rho \frac{V^2}{2}$, is known as the "dynamic pressure." Constant C is less for wide than for narrow members and less for streamlined than for blunt surfaces.

Note.—Written comments are invited for publication; the last discussion should be submitted by February 1, 1953.

The resultant of the steady wind force usually does not act exactly in the direction of the wind but is deflected, often to a considerable extent, by a force component which acts at right angles to the direction of the wind. In the field of aerodynamics this is known as the lift. The component in the direction of the wind is known as the drag.

As a rough, preliminary conception, the varying pressures over the surface of a body in a wind stream may be regarded as the forces which must by applied to the windstream to change its direction, acting in general toward the center of curvature of the streamline. Since the air pressure within the undisturbed mass of the wind stream remains constant the added force causing the curvature is supplied by the body as an added pressure at the leading edge, deflecting the stream away from it and contributing to the drag, and a reduced pressure where the curvature must be reversed in order for the wind element to follow the surface of the body thus contributing to the lift (see Fig. 1).

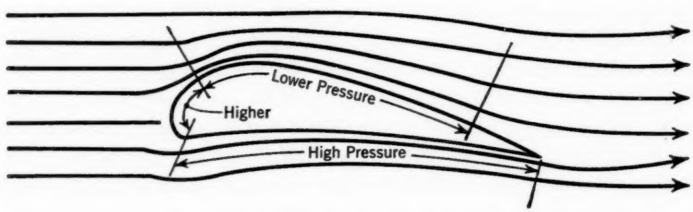

FIG. 1.—STREAMLINES AROUND A WING SECTION

Actually the situation is not so simple as conceived above due to the action of viscous frictional forces which retard a layer of wind near the surface, known as the boundary layer. A discussion of the boundary layer and its effects on the flow and forces can be found in any reference book on aerodynamics. It suffices here to state that it is only in the boundary layer that viscous effects are important.

The velocity varies from point to point in accordance with the streamline pattern associated with the displacement of the flow by the body and its boundary layer and wake. Outside the boundary layer and wake, the pressure and velocity are related by Bernoulli's law, which states that the difference in pressure between two points in a fluid is proportional to the difference in the squares of the velocities at the two points, the direction being such that an increase in velocity means a decrease in pressure. Obviously the shape and position with reference to the direction of the wind are very important in determining the relative amounts of lift and drag. In general the higher velocities and lower pressures tend to occur along the streamlines following the longer paths around the body unless they involve too sharp changes in direction. Fig. 1, which will be recognized as a wing section, is an example of a body producing high lift and low drag. Due to the shape and angle of attack the pressure over the upper surface is much lower than that over the lower surface. Lift may be obtained on objects of less regular shape and may, of course, be up or down. The resultant force is likely to fall to one side of the center of

gravity of the object and of its support, resulting in a torque tending to produce rotation.

If the curvature of the surface is excessive, boundary-layer effects cause the air near the surface to move from a higher pressure region to a lower one, and at times even reverse direction, causing the growth of eddies or vortices which break away periodically producing a highly turbulent wake. These phenomena are usually associated with a loss of lift and an increase of drag.

Of course a blunt, angular bridge structure has little of the smooth curved surface of an airplane wing, and turbulence is the dominant characteristic of the wind stream around a bridge. Nevertheless, there is a pattern of flow curvature and of moderate velocity and pressure differentials characteristic of an envelope of significant parts of the bridge cross section, supplemented by similar but more pronounced effects over limited areas where stronger and smoother flow occurs locally. These forces would have little influence on vibration of the bridge but for the fact that they have a periodic fluctuation or pulsation which can be in resonance with a natural mode of vibration of the bridge and thus accumulate energy and build up the amplitude. It is startling to discover how small a periodic force can be (of the order of 1 lb per ft of bridge) and still be capable of causing a substantial amplitude of vibration (1a).[1]

It has been pointed out that the pattern of streamlines about a body is determined, not only by the shape and attitude of the body, but also by the presence of the boundary layer and wake. Because of this fact the viscosity can have an important influence on the velocity and pressure field and hence on the forces. Associated with this effect is what is commonly called scale effect, which arises from the fact that the inertial or pressure forces are proportional to the square of the velocity while the viscous forces are proportional to the first power of the velocity and inversely proportional to the linear dimensions of the body so that the ratio of inertial forces to viscous forces will not be the same on a scale model as on its prototype (both in the same fluid) unless the velocity scale is distorted. The type of section, nature of the phenomenon under investigation and the model scale determine whether or not such distortion is desirable, permissable or practicable.

The ratio of the inertial forces to the viscous forces depends upon density, viscosity, velocity and length, and is expressed by a dimensionless scale parameter known as the Reynolds number denoted by R. Thus

$$R = \frac{V\,L}{\nu} \qquad \text{(2)}$$

where V is the velocity, L is a convenient linear dimension of the body and ν is the ratio of viscosity to density called the kinematic viscosity $\left(\frac{1}{\nu} = 6380 \frac{\text{Sec}}{\text{ft}^2} \text{ at } 15° \text{ C and sea level}\right)$.

Two geometrically similar bodies will in general have different force coefficients (such as C in Eq. 1) if the Reynolds number is different. In other words

[1] Numerals in parentheses, thus: (1a), denote corresponding items in the list of references, Appendix I.

they are geometrically similar but dynamically dissimilar. "Dynamic" refers here to the air and not to the model. This is an important consideration in model testing, for the model must show what the prototype will do. If a model is made geometrically similar to the prototype in all external detail and the Reynolds number under test conditions is made equal to the Reynolds number of the prototype, then both geometrical and dynamical similarity of the windstream exist. According to the foregoing considerations the model test should then give the force coefficients of the prototype. However, the turbulence in the air stream of most wind tunnels also has an effect like Reynolds number. Fortunately it has been found that wind-tunnel-turbulence effects are small when the Reynolds-number effect is small. It therefore becomes necessary to conduct a test over a range of Reynolds numbers to determine the effect of Reynolds number.

In the case of flow over blunt and comparatively sharp-edged objects like bridge sections, Reynolds-number effects are not encountered to any significant degree, as proved by comparative model tests at different scales and velocities, and illustrated by smoke stream studies of the paths of the wind (2a) (3c) (3d) (3f).

It is apparent that above some large value of the Reynolds number (that is, the ratio of inertial force to viscous force) the viscous forces may have small effect and a further increase in R results in no change in force coefficients. In the case of flow around blunt and sharp-edged objects, even though the Reynolds number may be low, the relative influence of the viscous forces may be subordinated by the turbulence and resultant loss of energy.

Naturally, the lift forces vary when the wind is angled upward or downward from the horizontal. Usually an upward angled wind (positive angle of attack) has a greater positive lift than occurs in a horizontal or downward angled wind. However, there are many shapes of an obstruction which show, paradoxically, a reduction in the lift when the wind has an upward vertical component, even to the degree that the upward moving wind may exert a net downward force (negative lift) on the obstruction. The reverse also can occur, a downward angled wind (negative angle of attack) causing a net upward force

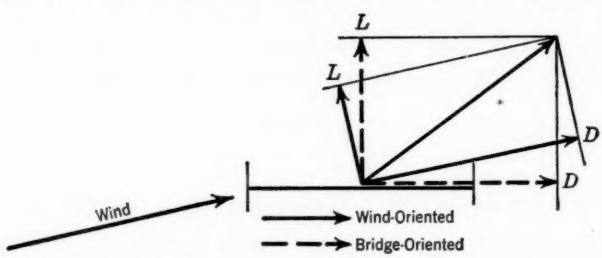

FIG. 2.—ORIENTATION OF LIFT AND DRAG

(positive lift). It will be recognized from the foregoing discussion that this can occur if the velocities are the greater across the surface more directly approached by the angled wind, especially if excessive turbulence destroys the lift action on the opposite surface.

When a model of a section having the characteristics described in the preceding paragraph is tested in a wind tunnel and the measured lift at different angles of attack is plotted against angle of attack the curve shows a negative slope, with lift decreasing for increasing angle over a certain range of angle of

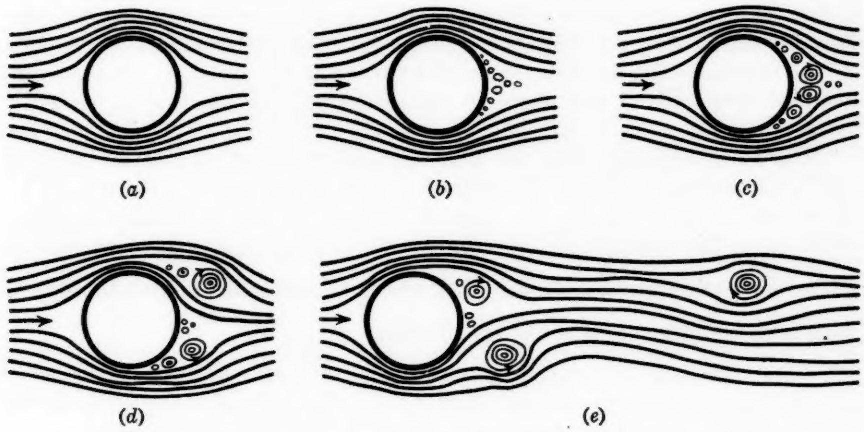

Fig. 3.—Vortex Development

attack and with positive slope over other parts of the curve (2b) (3g). The practice is to plot the coefficient of lift C_L (corresponding to C in Eq. 1) rather than actual lift. Also, for the study of bridge sections the measured values should be corrected to orient them to the horizontal axis of the bridge rather than to the direction of the wind (See Fig. 2). Curves of this type are shown subsequently in Fig. 6. This negative slope of the lift curve is the basis for one hypothesis explaining in part, the oscillation of suspension bridges in the wind. The action is described in clear detail by J. P. Den Hartog as an explanation of the "galloping" of transmission lines (4a). For want of a better name it is referred to as the "Negative Slope Theory." Its application to the oscillation of a bridge is discussed in Art. 6.

(B) *Vorticity.*—In an earlier paragraph mention was made of the formation of eddies or vortices due to the flow of the fluid (liquid or gas) in response to the pressure gradient along the surface of the obstruction. It will be profitable to examine this phenomenon further.

It is to be expected that a difference in pressure along a surface will cause a flow of fluid from the higher toward the lower pressure area (unless the surface is so well streamlined that the viscous or turbulent stresses in the boundary layer prevent the flow) and this is borne out by a series of photographs made by L. Prandtl showing the streamlines around a cylinder which was moving at a uniform velocity through the fluid (5a). Immediately after the start the streamlines show the theoretical "potential flow," smooth curving lines with dead spots or "stagnation points" centered on the leading and trailing edges (Fig. 3a). This condition continues at the leading edge but the photographs show that a moment later a mass of turbulent fluid accumulates at the center of

the trailing edge (Fig. 3b) and subsequent photographs show this mass growing and spreading and crowding the streamlines farther away from the trailing edge as shown in Fig. 3c. At the same time this mass of fluid is seen to take on twin whirling motions due to the viscous drag from the passing windstream. Finally the enlarging eddy itself becomes a major obstruction to the flow and is torn away by the forces acting upon it (Fig. 3d) and carried away in the fluid stream, moving at a velocity somewhat less than that of the fluid (Fig. 3e). (Fig. 3 illustrates the development of the flow around a cylinder. It occurs in modified form around any shape of obstruction except one that is perfectly "streamlined.")

It is to be expected that the time required to build up and discharge a vortex will increase with the diameter, d, of the cylinder but will be reduced by an increase in the velocity, V. This is indeed the case as verified by tests which show the time interval between vortex discharges may be expressed as a function of the ratio, $\frac{d}{V}$. V. Strouhal, who first determined this relationship in 1878 expressed the frequency of the discharge as

$$N = \frac{S\,V}{d} \qquad (3)$$

in which S is known as the Strouhal number and is about 0.2 for a cylinder. It varies considerably with the shape of the obstruction (between approximate limits 0.14 to 0.30) and for streamlined or partially streamlined obstructions it is influenced by the Reynolds number (3c) (6) (7).

Not only may one anticipate a regular time interval between these vortex discharges, but one may expect also that if one vortex is whipped off by one side of the fluid stream the next vortex, already forming, will accumulate and be discharged from the other side, rotating in the opposite direction, and that thereafter the vortices will be discharged alternately as shown in Fig. 3e. This also has been demonstrated by tests, and Mr. von Kármán has formulated the mathematical theory for the vortex path, frequency, spacing, and velocity and the forces acting on them and on the obstruction (5b) (9). The pattern of alternating vortex discharge in the wake of the obstruction is known as the Kármán trail.

The fluid motion which causes the alternating vortex discharge also causes inertial forces to react against the obstruction thus subjecting it to alternating pressures. The strength, point of application, direction and frequency of these forces are determined by the stream velocity and the form of the obstruction. They may cause major fluctuations in the lift and relatively smaller variations in the drag. Clearly, these can be a cause of oscillatory motion of a body such as a paddle drawn through the water or a suspension bridge in the wind. This explanation, referred to as the "Vortex Theory," will be discussed in Art. 7.

It is well to note a distinction between two types of vortex discharge—(a) the starting vortex which is initiated following any new wind condition (velocity or direction) and is discharged after a distinctive period of time during which

the potential flow is modified while the first vortex builds up, and (b) the Kármán vortices which follow the starting vortex in rhythmic alternation (5c).

2. Natural Vibrations of a Suspension Bridge

Suspension bridges, in common with all elastic structures, vibrate at definite frequencies determined by the ratio, $\sqrt{\frac{k}{m}}$, in which k is the spring constant and m is the mass, both expressed in consistent units. In common with all beams they have an infinite number of degrees of freedom and can vibrate in an infinite number of displacement forms, each at its own distinctive frequency. The bridge can oscillate in a pure vertical motion or a rotation about a longitudinal axis and with a single loop over the entire span (the fundamental mode) or with any number of loops although the designer is seldom concerned with modes with more than three of four loops in the main span. (The original Tacoma Narrows Bridge and its full model in the wind tunnel vibrated (2c) (3h) with as many as nine loops but these higher modes were rare and their amplitudes were small.) Fig. 4 shows the forms of several modes of vibration of the original Tacoma Narrows Bridge and its full model. (The broken lines and numerals in parentheses, in Fig. 4, represent the modified condition with the mid-point of the main cable tied to the suspended structure.) Similar forms have been observed on other models (3i) (3j).

If the mode has an even number of loops one of the stationary or node points is at mid-span and the wave form is asymmetrical about this point. Also the waves are balanced vertically about the normal position which makes it possible for the main span to vibrate while the side spans are still. On the other hand, the fundamental mode cannot occur unless the side spans move symmetrically and in the opposite direction to the main span movement (except for the small amplitudes which can be permitted by the stretching of the cables). This is generally true of all the symmetrical modes (with odd numbered loops) except that as the number of loops increases it becomes possible for their lengths to vary so that the areas under the loops above and below the normal position are nearly equal and the motion is approximately balanced on the main span and therefore only slight motion of the side span is necessary, as illustrated by mode type 8 of Fig. 4.

Most of the spring constant (that is, the stiffness) of a suspension bridge is due to the resistance of the loaded cable. The stiffening truss, added to reduce local bending under concentrated loads, adds only moderately to the overall stiffness and to the frequency of vibration. Its influence increases for the higher modes with their sharper curvature. For example, the 33-ft truss of the new Tacoma Narrows Bridge contributes about 20% to the frequencies of the fundamental mode and the first asymmetric mode (two loops) but (3k) doubling the truss stiffness would increase the frequencies of these modes only about 12%. For the second symmetric mode (two nodes and three loops on the main span) the truss stiffness contributes 30% to the frequency and doubling it would increase the frequency another 20%. Making the stiffening truss continuous at the towers would increase the frequency only slightly (1b).

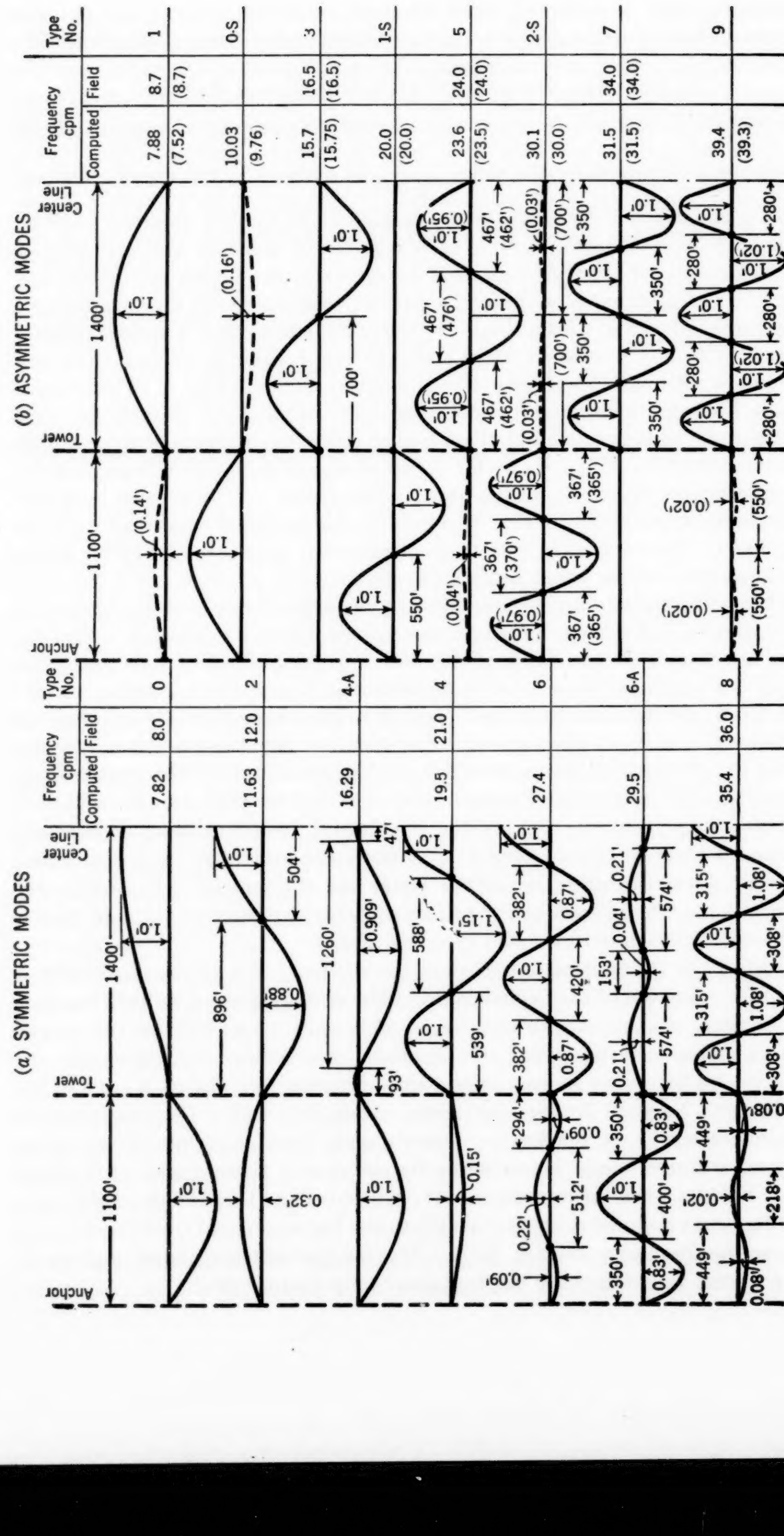

FIG. 4.—MODES OF VIBRATION, ORIGINAL TACOMA NARROWS BRIDGE

For torsional oscillation the motion is substantially the same as for the corresponding vertical mode except that the two sides of the bridge move in opposite directions—that is in opposite phase. The cable and truss offer the same resistance to the torsional as to the vertical displacement and the equivalent spring constant, k, is the same. However, the effective mass is less for the torsional than for the vertical motion, being that mass which, if concentrated at the center line of the truss, would have the same mass moment of inertia about the axis of rotation as does the actual structure. Thus if m is the mass of the cables, plus the mass of the suspended structure, b is the width center to center of trusses, and r is the radius of gyration of the entire structure, then the effective mass is

$$m' = m \left(\frac{2\,r}{b}\right)^2 \qquad (4)$$

For most suspension bridges the torsional frequencies are approximately 25% greater than the corresponding vertical frequencies.

The foregoing statement is based on the assumption that there is only one phase of lateral bracing. With lateral systems in the planes of both the top and bottom chords, the resistance of the structure to torsional motion is completely changed. The four-sided system of trusses behaves as a tube under torsion and its resistance depends upon the depths of the four trusses and the areas of their diagonals instead of the depth of the vertical trusses and the areas of their chords. The high torsional rigidity of such a four-sided truss system contributes the major resistance of the span to torsional displacement and thus increases materially the frequencies of the torsional modes. For example, in the new Tacoma Narrows Bridge, the addition of the second plane of laterals (in the plane of the bottom chord) increased the frequency of the first and second symmetric torsional modes by 128% and 176% respectively (1c).

In most suspension bridges the tower offers comparatively little resistance to a force at the tower top tending to cause bending parallel to the bridge axis and therefore has little effect on the vertical modes of vibration. In the case of a symmetric torsional mode, however, the tops of the two columns of each tower move in opposite directions because the two cables move in opposite phase and thus the tower is subjected to torsion. The torsional rigidity of modern bridge towers of box or cellular section with substantial bracing is considerable and does influence the frequency as well as the wave form of the symmetric torsional modes (1d) (3l).

In the first asymmetric mode the cable tends to straighten over one half span and take on greater curvature over the other half. This causes the midpoint of the cable to move back and forth in a longitudinal direction. If a shear tie is introduced between the truss and the midpoint of the cable then the entire suspended structure must participate in this longitudinal movement and the frequency will be reduced due to the additional mass moved. If, with the center tie, the suspended structure is anchored at one tower the first asymmetric mode will be inhibited unless the tower tops and side spans also oscillate as shown in Fig. 4 (Type O-S). The first asymmetric torsional mode will be inhibited even without end anchorage because the two cables will pull

in opposite directions on the suspended structure. (The couple resulting from these opposing forces at opposite sides of the roadway tends to produce a reversed bending of the suspended structure in the horizontal plane but usually this tendency is slight.)

The bridge offers considerable resistance to a lateral oscillation and there is little evidence that the wind tends to cause such motion (a strong wind does cause a non-oscillating horizontal deflection). A small lateral oscillation accompanies torsional oscillation if the axis of rotation is raised above the deck by some means such as diagonal stays across the roadway between each cable and the opposite truss (with a strut between cables at the points of connection of the stays). This causes the cables and suspended structure to oscillate laterally in opposite directions and tends to increase the frequency.

For a suspension bridge the formula for frequency—

$$N = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (5)$$

—becomes somewhat complicated. Considering a bridge with a negligible stiffening girder, the spring constant comes entirely from the resistance of the loaded cable to displacement from its dead load position. This resistance is of course, proportional to the dead-load tension in the cable which in turn is proportional to the dead load. Thus it is that an addition to the dead load which increases m causes a proportionate increase in k so that the frequency remains unchanged by the additional weight. (This statement is not precise since the stretch of the cable permits a yield, usually small, in addition to the effect of cable displacement.)

The dead-load tension in the cable is also inversely proportional to the sag. Therefore the spring constant varies inversely with the sag and the frequency varies inversely with the square root of the sag.

If a substantial stiffening truss is added then an addition to the spring constant is made which is independent of the mass, and the frequency becomes more responsive to a change in the mass. The loaded cable and the truss both offer increased resistance to multiple loop distortion (per unit displacement) and therefore the frequency increases by steps for the higher modes.

If two springs act in parallel to carry a load, then each carries but a part of it and the deflection will be less than if either sustained the load alone. Their resistances are additive and the combined spring constant is the sum of the individual spring constants. On the other hand, if they act in series the total load acts on each of them and their yielding is additive. With series loading the reciprocal of the combined spring constant is the sum of the reciprocals of the individual spring constants; that is, the combined spring constant is less than either individually. It is apparent from the frequency formula, then, that if an elastic element is added to the bridge in such a way as to increase the resistance to displacement it will tend to increase the frequency whereas if it is added in such a way as to increase the yielding under loading it will tend to reduce the frequency. Thus the addition of a stiffening truss increases the frequency whereas increasing the sag increases the yielding

and reduces the frequency. This simple principle makes it easy to form a qualitative judgment of the influence of almost any elastic element of the structure.

3. Factors Affecting Natural Vibrations

With the preceding article as a background the principal factors which influence the natural vibrations of suspension bridges may be summarized. (It will be seen in the next article that it is desirable to keep the frequency as high as economically feasible.)

(A) *Sag.*—The frequencies of all the modes vary inversely as the square root of the sag. If other resisting elements are so weak that the spring constant is determined by the cable force alone then the frequencies of the asymmetric modes depend only on the sag (substitute the value of μ given by F. C. Smith and G. S. Vincent (3m), Members, ASCE, and solve for ω) and those of the symmetric modes on only the sag and the ratio of the side span to main span length (1e) (3n). (Except the symmetric mode shown in Fig. 4, Type 4-A, which can occur only by virtue of cable stretch (3 o).)

(B) *Side Span Length.*—The asymmetric modes are not affected by side span length. Shortening the side spans increases the frequencies of the symmetric modes and also distorts their form. If the side spans are unequal in length, the perfect symmetry of the symmetric modes will be destroyed but the modes will not be inhibited.

(C) *Unloaded Back Stays.*—Unloaded back stays represent the ultimate reached in shortening the side spans. The effects are described above, though more pronounced.

(D) *Moment of Inertia.*—An increase in the moment of inertia increases the frequency and alters the wave form, although the effect is only moderate within practical limits. On longer spans increasing the moment of inertia several-fold beyond the requirements for limiting grade change produces only a small increase in frequency (1f).

(E) *Torsional Rigidity of Suspended Span.*—If the lateral resistance to the horizontal wind force is provided by two lateral systems in the planes of the top and bottom chords rather than concentrated in a single system in one plane, the resulting four-sided truss system will have a torsional rigidity which will double or treble the torsional frequency (1g). (This double system does present an added design problem for certain positions of the live load.)

(F) *Weight.*—Adding weight has little effect on frequencies, except where the truss contributes a large part of the total stiffness (1h). However, an increase in weight increases the total energy of vibration at any given amplitude without affecting the aerodynamic forces exciting motion and therefore reduces the relative effectiveness of these forces. This will be further discussed in Part II of this report.

(G) *Center Ties.*—Center ties are quite effective for preventing the first asymmetric torsional mode (which destroyed the original Tacoma Narrows Bridge). If combined with longitudinal restraint of the stiffening truss at one tower they will strongly oppose the first asymmetric vertical mode but will

produce heavy stresses under asymmetric live loading. They have no effect on symmetric modes because these involve no relative longitudinal movement of the suspended structure and the midpoint of the cable.

(H) *Diagonal Stays.*—Diagonal stays from the tower tops to intermediate points on the truss tend to inhibit a mode unless the point of attachment to the truss happens to be at a node point. If a mode is forced in spite of the stays, they increase the frequency moderately due to their added resistance. In wind tunnel tests a single system of stays (from each tower top to one point on the side span and one point on the main span) delayed the excitation of the mode until a higher velocity was reached after which the stays had little noticeable effect, except to alter the frequency (3d) (3p) (10a). Multiple stays are more generally effective. These, however, make the structure highly indeterminate and susceptible to alteration in elastic properties when affected by temperature change. Stays from the tower at about deck level to points on the cables out on the span have similar dynamic effects but their temperature influence is greater because the stays oppose the main cables.

(I) *Cross Road Stays.*—Stays extending from each cable diagonally across the roadway to the opposite truss and supplemented by struts between the cables at the point of attachment of the stays have a strong effect on torsional modes unless they are near the node point. They do not prevent the motion but move the center of rotation upward so that the cable must move laterally in one direction while the truss moves in the opposite direction and the frequency is considerably increased due to the added resistance thus encountered (3d) (10a).

(J) *Towers.*—The towers must be flexible or hinged to permit longitudinal motion under changing live load, and therefore offer very little resistance to the vertical modes; however, their torsional rigidity is considerable and they have a very noticeable effect on symmetric torsional modes (1d) (3e). Nevertheless, only a small additional benefit is gained by even a considerable increase in the metal necessary to provide for the stability of the tower itself.

(K) *Hangers.*—The hangers act in series with the cables to sustain the loading and therefore their stretch increases the yield and tends to reduce the frequency but the effect is negligible.

(L) *Cable Stretch.*—Cable stretch makes possible one peculiar vertical and a similar torsional mode on almost any bridge and completely controls its frequency and wave form (3o). Cable stretch also has a modifying effect on other symmetric modes, particularly if the back stays are not loaded. If the displacement form of the mode is approximately balanced about the normal position, then the additional stress due to vibration is small and the effect of cable stretch is negligible.

(M) *Lateral Stiffness.*—Lateral stiffness does not appear to be involved in the aerodynamic vibration of suspension bridges except in combination with cross road stays which force lateral displacement of the structure. This is not to deny the possibility that a bridge may be designed with such lateral flexibility as to bring about objectionable oscillation.

4. General Behavior of a Bridge in the Wind

(A) *Constant* $\frac{V}{N\,b}$ *-Ratio.*—The dominant characteristic of the behavior of a bridge in the wind is the presistent and consistent relation between the frequency (N, cycles per second) of the vibration of the structure and the velocity (V, feet per second) of the wind which causes it. It is convenient to divide this ratio by b, the width of the bridge in feet (a constant for any given bridge) because this makes the ratio non-dimensional and therefore applicable without alteration to a bridge or any scale model of the bridge. The significance of this ratio is further revealed by the fact that it shows up as one of the most important parameters in theoretical analyses dealing with the forces and motions involved in aerodynamic vibration (see any reference on aeroplane or suspension bridge vibration (11)).

It is helpful to examine this ratio for its physical interpretations. First, $\frac{b}{V}$ equals the time required for an element of the wind stream to cross the bridge and $\frac{1}{N}$ is the period in seconds of a cycle of the bridge vibration; therefore $\frac{V}{N\,b}$ is the ratio of the period of vibration to the time required for a particle in the wind to cross the bridge. Similarly, $\frac{V}{N}$ is the distance a particle in the wind travels during one cycle of the vibration and $\frac{V}{N\,b}$ is the ratio of this distance to the width of the structure. Again $N\,b$ is the velocity at which a particle must move to cross the bridge in one cycle and $\frac{V}{N\,b}$ is the ratio of the wind velocity to that velocity. The ratio $\frac{V}{N\,b}$ expresses the wind velocity with b as the unit of distance and the period, $1/N$ as the unit of time. In all of these conceptions the $\frac{V}{N\,b}$ -ratio relates the motion of the bridge in any of its modes to any periodic phenomena which occur in the wind stream such as the Kármán vortex discharge.

(B) *Character of Motion.*—Any form of wind-actuated oscillation of a bridge can be classified as restricted or catastrophic. Oscillations of restricted character will start at a definite wind velocity and will increase in amplitude as the velocity increases until a maximum is reached (2d) (3c) (3d) (3e) (10b). With further increase of velocity the amplitude decreases and, at a slightly higher velocity the motion ceases. The $\frac{V}{N\,b}$ -ratio at which the motion starts is known as the lower critical; that at which the motion ceases, as the upper critical. Each of these has a substantially constant value for all vertical modes and a different value, also approximately constant, for all torsional modes. However there are often weak appearances of a mode at approximately a submultiple of the $\frac{V}{N\,b}$ -ratio which characterizes its principal appearance.

In the case of any catastrophic motion, the oscillation begins at a definite critical $\frac{V}{N\,b}$-ratio but with increased velocity (or in some cases without increase in velocity) the oscillation builds up rapidly and there is no upper critical velocity.

Factors which influence the critical $\frac{V}{N\,b}$-ratio can be listed as follows (3c) (3d) (3e) (11):

(a) The shape is perhaps the most important factor.

(b) The angle of attack alters the path of the wind with respect to the section and modified the $\frac{V}{N\,b}$-ratio.

(c) The type of motion whether vertical or torsional.

(d) For truss-stiffened bridges and flat plates placed horizontally in the wind, the wind-forced motion is usually a combination or "coupling" of the vertical and torsional types of motion which are forced to a common frequency by the action of the wind. This occurs at a distinctive $\frac{V}{N\,b}$-ratio.

(e) Damping increases the critical ratio of a catastrophic oscillation and tends to increase the lower and decrease the upper ratios of a restricted motion.

(C) *Behavior of a Girder-Stiffened Section.*—In all tests of oscillating models of girder-stiffened sections except extremely shallow girders the motion has been either a pure vertical mode or a pure torsional mode at substantially the natural frequency of vibration (3c) (3e). For such a section the $\frac{V}{N\,b}$-ratios and the character of oscillation (restricted or catastrophic) are influenced primarily by the ratio, d/b, of the depth of the girder to the width of the bridge. Details such as girder flanges, stringers, and curbs, and the position of the deck with reference to the girders also have important modifying influences. This influence of details is not always predictable.

It is characteristic of girder sections that they may oscillate at restricted amplitudes at moderate wind velocities and with a variety of successive modes and corresponding frequencies as the velocity changes. In general, depending upon the shape, both vertical and torsional modes may reappear at higher velocities (but at their characteristic frequencies), and some of these may be catastrophic.

As the $\frac{d}{b}$-ratio decreases below about 1/7 the girder-stiffened section shows increasing evidence of the coupled type of oscillation described in the next paragraph.

(D) *Behavior of Truss-Stiffened Sections.*—All oscillations that have been observed on models of truss-stiffened bridges and the observed small motion of the Golden Gate Bridge as well have been of the coupled type (3d) (3e) (11). Often the mode appears to be almost a pure torsion but the frequency is reduced toward that of the corresponding vertical mode. In many cases

(including all of the catastrophic motions observed), the vertical mode component has sufficient amplitude to shift the axis of rotation considerably to the windward from the center line of the structure. In some cases the axis of rotation has been practically at the windward truss or even well beyond it. It appears very probable that the oscillation which damaged the Menai Straits Bridge (3q) in 1839 was coupled since the leeward side of the bridge deck was largely destroyed while the windward carriage-way was only moderately damaged. Also the destructive oscillation of the Brighton Chain Pier was described as consisting of both "an undulatory motion along the road" and "oscillating motion across the roadway (3q)."

It is characteristic that coupled types of oscillation occur at considerably higher velocities than do the pure vertical and torsional types of the girder sections. It is apparent also that the aerodynamic forces on the truss-stiffened section are determined primarily by the deck and truss members adjacent thereto. In model tests the removal of the bottom chord and web members of a deck truss section has affected the motion only slightly. These members are beneficial aerodynamically in that they add a little to the aerodynamic damping.

Since coupled motion requires a blending of vertical and torsional motions in a manner to be mutually exciting under wind action, it depends upon the elastic factors affecting the relative frequencies of the vertical and torsional modes (11). The use of two planes of laterals as described in the preceding article increases the torsional stiffness and thus throws the vertical and torsional frequencies of corresponding modes so far apart in many cases that the critical velocity of coupling and flutter is increased beyond any probable wind velocity.

5. Value of Theories

The fact summarized in the preceding articles have shown how the bridge oscillation problem is related to other, more familiar problems and have suggested theories explaining what happens. It is helpful to state these theories, even with their limitations, because they help to give a clearer conception of causes and effects and possible cures. Moreover a theory is essential to any purely mathematical solution of the problem.

6. Negative Slope Theory (3r) (4a) (10) (12)

If, from any cause, a bridge is moving downward while a horizontal wind is blowing the resultant wind with reference to the bridge is angled upward (positive angle of attack) as illustrated by Fig. 5.

If the lift coefficient, C_L, as measured in static tests shows a variation such as illustrated by curve A in Fig. 6 then, for moderate amplitudes, there is a wind force acting downward on the bridge while the bridge is moving downward. The bridge will therefore move to a greater amplitude than it would without this wind force. However, the motion will be halted and reversed by the action of the elastic forces, whereupon the vertical component of the wind reverses, the angle of attack becomes negative and the lift becomes positive tending to increase the amplitude of the rebound. As the bridge oscillates with increasing amplitude its vertical velocity, v, also increases and therefore the

maximum positive and negative values of α increase until a part of the motion occurs in the angle range where the lift opposes the motion—that is, where the slope of the lift coefficient is positive. Finally an amplitude is reached at which the net energy derived from the wind action in a cycle is just sufficient to

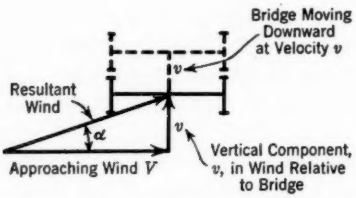

FIG. 5.—ANGLE OF ATTACK, α, INDUCED BY BRIDGE MOTION

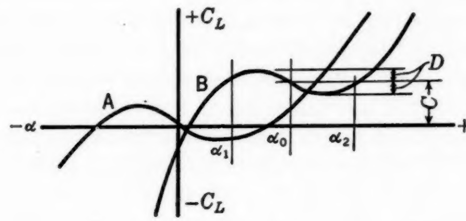

FIG. 6.—TYPES OF LIFT CURVE

offset the energy loss per cycle caused by frictional or damping forces. Thereafter, the wind velocity remaining constant, the bridge oscillates at a steady state amplitude. If, now, the wind velocity, V, is increased then (see Fig. 5) a corresponding increase in the velocity of vibration, v, must take place if the maximum range of α is maintained at its steady state value. This increase in v for a given mode of vibration can take place only if the amplitude of vibration increases since the frequency remains constant. Thus it is seen that with an increasing velocity the amplitude will increase indefinitely or until the bridge is destroyed.

If the region of negative slope occurs elsewhere on the graph than at the origin (see curve B in Fig. 6) the situation is only slightly altered. If horizontal and vertical lines are drawn through the approximate midpoint of the region of negative slope the lift acting on the stationary bridge will be C if the windstream is turned upward at an angle, α_0, from the horizontal. If the bridge oscillates the effective angle of attack will range, say, from α_1 to α_2 and the bridge will be acted upon by the steady lift, C, plus or minus a varying lift, D. The steady lift will oppose the downward motion and assist the upward motion and its net energy contribution will be zero. However, during the downward motion the effective angle of attack will be increased as previously described and the force D will be negative and will assist the motion. Also as the bridge moves upward and the effective angle is diminished, D becomes positive and assists the upward movement. For the lift graph shown as A, Fig. 6, the maximum effect occurs in a horizontal wind. For that marked B the action is the same except that the maximum effect occurs in a wind angled upward an angle α_0.

A similar though more complicated situation may occur with respect to a torsional or twisting motion of the bridge. If the resultant wind force acts eccentrically with respect to the elastic and inertial forces acting on the bridge it tends to rotate the structure about its longitudinal axis. There are sections of such shape that, with the change in angle of attack resulting from this rotation, there is an increase in the torque tending to cause the rotation. In such cases static tests reveal a characteristic reversal in the slope of the curve

for the moment coefficient, C_M, when plotted against angle of attack. (The actual sign of the slope depends upon the sign convention for moment and angle.) The action is complicated, however, by the fact that in the case of torsional oscillation, the instantaneous angle of attack is a function of both the instantaneous position of the section and its instantaneous velocity. Also the velocity varies from zero at the axis of rotation to its maximum value at the edges and its direction is opposite at the two sides of the structure.

The negative slope theory is not in full agreement with the observed behavior of bridges and models, the most important defects being:

(a) It does not explain the fact that an oscillation of a given frequency may begin at a certain wind velocity and increase in amplitude as the velocity increases but be stopped by the action of the windstream if the velocity increases further. In such a case the rate of decay is more rapid than in still air.

(b) It does not explain the excitation of structures for which the slopes of the lift and moment coefficients are found to be positive (10b).

(c) It leaves to chance the initial motion which gives rise to the initial change in effective angle of attack. This is not a serious deficiency since this initial displacement, perhaps on only a small part of the bridge, can be explained by wind gusts.

7. Vortex Theory

The vortex theory attributes the aerodynamic excitation of a suspension bridge to the action of periodic forces having a certain degree of resonance with a natural mode of vibration of the bridge. The alternating vortices, moving across the bridge with a rolling motion at about 80% to 85% of the speed of the wind (7) give rise to moving zones in which the wind velocity adjacent to one surface of the bridge is materially reduced, in contrast to the increased velocity next to the opposite surface. (Mr. von Kármán found 0.86 for a circular cylinder and 0.80 for a flatplate normal to the wind (9).) By Bernoulli's Theorem the pressure is the greater over the zone of reduced velocity. This is the source of the periodic force. The frequency of the vortex discharge is bound to the wind velocity by the Strouhal number. The wind velocity therefore determines the degree of resonance.

The action described above has been visualized through the use of smoke streams over a model in the windstream (3c). With the wind velocity a little above the critical and the model at rest, a weak pattern of alternating vortex discharge can be seen with a frequency, perhaps some multiple 2 to 4, of the frequency of the vibration of the model which is about to begin. These are Kármán vortices and their frequency corresponds to a reasonable value of Strouhal number (about 0.145 to 0.186) for a horizontal girder section (3c). The vortex forces, applied at a multiple of the natural frequency of the bridge, could not build up the motion of the latter if both the forces and the bridge motion were of simple harmonic variation. The forces would add energy to the bridge oscillation during part of the cycle and remove an equal amount of energy during the remainder. However, there is ample evidence that these forces are nonlinear in relation to the bridge movement. With nonlinear forces the slower bridge oscillation is built up by sub-harmonic excitation (4b).

As soon as the oscillation becomes distinct it is observed that the vortex action alters and only two vortices are discharged per cycle. One leaves the upper edge of the windward girder when the model starts down and the pressure zone of this vortex moves across the deck while the downward movement continues. The other leaves the lower windward edge when the model starts up and causes a similar upward acting pressure zone during the upward movement. These vortices are distinct, of large diameter and vigorous. They are regarded as "starting vortices," (5c) each being initiated by the change in angle of attack caused by the reversal of the motion of the model. They are enlarged due to the receding of the edge from which they are shed as well as the increasing effective angle of attack. The time required for their formation also seems to increase. This enlargement and slowing of the vortex discharge when oscillation of the obstruction occurs has been noted by previous investigators who found that the frequency decreased with increasing amplitude (7) (8). Thus when the increased velocity is "paced" more or less by increasing amplitude there is a limited velocity range within which the vortices can retain their frequency of two vortices per cycle of bridge oscillation. As the upper critical velocity is approached, however, they move across the section more rapidly until they act only during a small part of the half cycle and abruptly lose their effectiveness. Then the amplitude decays, the motion of the bridge "loses control" of the vortex action and the Kármán vortices reappear at the higher frequency dictated by the increased wind velocity. At that point the frequency of vortex discharge may have increased approaching resonance (usually sub-harmonic) with a higher natural mode of the bridge so that the higher mode is built up displacing the original. This process continues while the wind velocity increases and the bridge oscillation progresses through higher and higher modes, both vertical and torsional, with one definite value of critical $\frac{V}{N\,b}$-ratio applying to all the vertical modes and a different value to the torsional modes. If, however, the bridge is subject to catastrophic oscillation one of the lower modes will reappear in catastrophic form at a higher $\frac{V}{N\,b}$-ratio (a multiple or nearly so of its earlier appearance). Section model tests of some sections at very low damping reveal a weak appearance of a mode at a lower velocity also. Some of these may be in resonance (not sub-harmonic) with the rate of the Kármán vortex discharge (3c).

It is seen that the vortex theory does account for the initial small amplitude of a given mode and also explains the selective action which limits each mode to a definite velocity range consistent with the observed $\frac{V}{N\,b}$-ratio.

8. Flutter Theory

When the first full model of the proposed design for the new Tacoma Narrows Bridge with solid deck and one lateral system was first tested in the wind tunnel late in 1943 a torsional oscillation developed but its frequency was 6% to 8% less than the natural torsional frequency of the model when excited in the same mode by hand (3d). Also the leeward side of the model

had a greater amplitude of motion than did the windward so that the axis of rotation was not on the longitudinal center line but upwind from it. In effect the oscillation was a combination of both the torsional and vertical modes of the same form and the frequency was intermediate between those of the two pure modes.

When this behavior was brought to the attention of Friedrich Bleich, M. ASCE, he suggested that it indicated flutter as experienced with the wings and other elements of aeroplanes, and applied to the suspension bridge the general theory previously applied to the aeroplane by Theodore Theodorsen and others. His theoretical analysis (1) was first submitted in a report to the Advisory Board in 1945. In a later report it was modified to take into account the additional effect of vortex forces. It was published (11) in October 1948. A similar analysis was later made (12) by D. B. Steinman, M. ASCE.

The pure mathematical theory of flutter applies to an infinitely thin flat plate placed horizontally in a horizontal wind. Such a plate is "stable" in the sense that it does not shed the disturbing Kármán vortex trial.

The mechanism of flutter depends upon the fact that the plate is supported so that it can move elastically in a vertical direction and in torsion about a longitudinal axis. The wind force causes a lift which acts eccentrically and therefore also induces a twisting moment. The resulting torsional movement alters the angle of attack so that the lift is increased which also increases the twisting moment. This chain action quickly becomes catastrophic provided the vertical and torsional motions can take place at the same coupled frequency and at an appropriate phase relation. This necessary condition is brought about through the influence of the wind. If such a plate is oscillated by external means while in a wind stream the frequency of the natural vibration is altered progressively by an increase in the wind velocity which reduces the frequency of the torsional mode and increases that of the vertical until, at some critical velocity, they have a common frequency. At this wind velocity any slight movement of the plate, whether vertical or torsional or mixed, takes on this common frequency thus satisfying the conditions for flutter, and develops rapidly into the coupled type of oscillation, the action being catastrophic, almost explosive.

The ideal, theoretical condition is never fully realized. The plate has a finite thickness and may have a blunt edge; and it does shed a vortex trail, which is accentuated if there is an angle of attack of a few degrees. The effect of this vortex action is to bring on vibration at a lower velocity than the critical velocity of flutter. This logical effect has been demonstrated by tests made on oscillating flat plates at the University of Washington for the purpose of verifying Friedrich Bleich's application of the flutter theory (3e) (11a). This vortex effect—that is, the reduction in critical velocity—becomes more pronounced if the thickness is increased. At the same time however, the flutter, brought in at a lower velocity, is less explosive and may require an increase in velocity before it becomes catastrophic.

Tests on oscillating section models of several truss-stiffened sections and of very shallow girder-stiffened sections show that such sections have the characteristic flutter behavior of a flat plate modified by the vortex discharge of

the blunt edges. One truss-stiffened section showed (3d) a coupled oscillation, with a weak vertical component and a frequency only a little less than that of the natural torsional mode, coming in at a velocity far below the critical value predicted by the flutter theory. This oscillation was vigorous but became catastrophic only after a considerable increase in velocity. As the velocity was gradually increased, approaching the computed critical velocity, the relative amplitude of the vertical mode component increased, shifting the axis of rotation to the windward. Also the frequency gradually fell farther below the natural frequency of the torsional mode. The computed critical velocity for flutter of an ideal flat plate having the mass and elastic properties of this model could not be reached in the wind tunnel but at the maximum velocity attainable the frequency was near the predicted value and the flutter was violently explosive; in fact, when the model was released at this velocity it was necessary to seize it again so quickly that an accurate measurement of the frequency was impossible.

9. Summary of Theories

It appears well established that both the vortex theory and the flutter theory apply to the behavior of suspension bridges in the wind and with many sections their effects are combined.

With a truss-stiffened section flutter is dominant but modified noticeably, often considerably, by the vortex action caused by the blunt edges (reduced critical velocity, less violent response). The flutter characteristics are controlled by the mass and elastic properties of the structure as a whole and the dimensions of the deck. The modifying vortex action is controlled by the thickness of the deck structure, as well as those truss members adjacent thereto. The positions of abrupt corners of curbs and stringers near the windward edge influence the degree of vortex action as does also the angle of attack. The influence of angle of attack is not symmetrical for plus and minus angles, probably because of the stringers and other unsymmetrical elements. There is little evidence that chords and web members at a distance from the deck have any but a damping effect. It is conceivable, however, that a section may have such dimensions and properties that the vortex discharge from a remote chord synchronizes with the motions caused by the dominant forces. Such a chance condition might result in unexpected activity of a bridge thought to be entirely safe on the basis of generalized design criteria. (However a condition of this kind could be readily detected by a simple series of tests on an oscillating section model of the adopted design.)

With a girder-stiffened section the vortex action is dominant. For all except unusually shallow girder sections the coupling of modes and alteration of frequency which characterize flutter cannot be detected or are very slight (3c) (3e) (11a).

The "negative slope theory" may possibly explain the excitation of the catastrophic vibration of a girder-stiffened section in the moderate to high velocity range in which the Kármán vortices in a cycle of bridge oscillation are so numerous and move so rapidly that the bridge cannot respond to their individual effects but does respond to the slower pressure changes controlled

by the changing angle of attack. This pressure pattern includes the combined or statistical effect of the stream of vortices traversing the portions of the bridge lying downstream from the sharp edge from which the vortices are discharged.

10. Energy and Damping

The picture of what happens when a suspension bridge oscillates in a windstream remains hazy unless the transfer, storage and absorption of energy are clearly understood. The oscillating bridge is a machine and as such its energy relationships are of major importance; the forces are of interest only if they become great enough (through the accumulation of energy) to threaten local failure. As in all complicated problems in vibration the analysis is made most easily by the energy method.

(A) *Nature of Damping Action.*—The total energy of vibration is proportional to the square of the product of the frequency and the amplitude. It is all in the form of kinetic energy at the instant when all parts of the structure pass at maximum velocity through their normal rest positions. At the instants of maximum displacement the structure is stationary and all the energy is stored as elastic energy in the strained cables and stiffening members and gravitational energy due to the changed elevation of the parts of the structure. Energy is lost due to air resistance, hysteresis loss in the strained material and friction between moving parts. If the wind forces are such as to cause oscillation of the bridge the net effect of the air is to add energy to the vibration. This effect can be treated mathematically as "negative damping." If this negative damping adds the same amount of energy per cycle that is lost due to hysteresis and friction then the energy of the vibrating system remains constant and the amplitude is maintained at its "steady state." The effect of a net loss or gain in energy is measured by a change in amplitude from one cycle to the next since there is no appreciable change in frequency.

The strength of a damping action is conveniently expressed by the "logarithmic decrement," δ, which is defined as the natural logarithm of the ratio of the amplitudes of two successive vibrations. For small damping the numerical value of δ is very nearly the ratio of the change in amplitude in one cycle to the amplitude of that cycle and twice this ratio, that is, $2\,\delta$, expresses the proportion of the energy of vibration, ψ, lost or gained per cycle. Instruments are available for measuring and recording conveniently the amplitude under any condition of positive damping or of excitation making it possible to determine the value of δ continuously. This indicates for a given vibration the rate of input or of dissipation of energy per cycle at all amplitudes and thus measures quantitatively the net work performed by all forces acting and obviates the necessity of determining the strength and lines of action of these forces and their detailed variation throughout the cycle. With these quantitative values determined under different conditions the individual effects of structural damping, still air damping and wind (positive or negative damping) can be computed for either model or prototype.

The various forms of damping differ widely in their effect on the rate of change of energy and amplitude and it is necessary to recognize their character-

istics when studying the influence of design features upon the magnitude of structural damping or studying the mechanism and quantitative effect of wind excitation. An analysis of these characteristics, as they bear on the study of wind-excited oscillation of suspension bridges, is assembled in Appendix II.

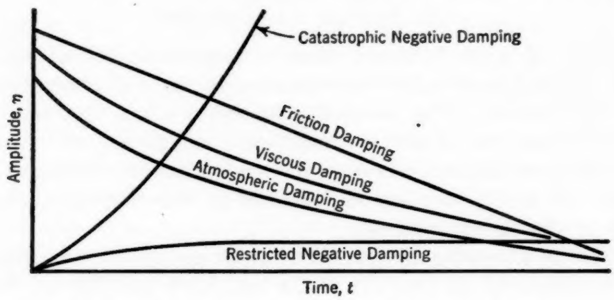

FIG. 7.—CHARACTERISTIC FORMS OF AMPLITUDE DECAY

The differences are illustrated to some extent in Fig. 7 which shows the rates of decay of amplitude with time and in Fig. 8 which shows the variation of the logarithmic decrement, δ, and the rate of change of energy, ψ, with the amplitude η.

(B) *Structural Damping.*—Varied information is available concerning structural damping. Hysteresis damping is quite low for steel (13) (14). Different investigators get different results depending upon the methods and type of specimen. The more rigorous tests generally indicate the lower values.

The structural damping of machines and structures varies widely. There is much and varied data available concerning bridges—usually simple spans (15) (16). However, none of this has much application to a suspension bridge because the stiffening members have neither the concentrated end reactions and accompanying frictional movements which provide most of the damping of beam or truss spans nor stress sufficiently high to produce material hysteresis effects. Friedrich Bleich has made a theoretical study of the sources of damping in a suspension bridge and correlated this with the results of damping tests on beams and trusses made by the Bureau of Public Roads (1i) (17). He has indicated the damping effect of sliding friction devices at the ends of the span and has also demonstrated how damping may be increased by designing to exploit sliding friction at the bearings of stringers on floorbeams.

It is hardly possible to secure quantitative data on the structural damping of a suspension bridge by means of model tests because the model details cannot be fabricated to reproduce the damping of the actual structure. This information must be obtained by tests on actual bridges representing various proportions of cable, truss and deck contribution to the damping. Preliminary studies to this end have been made by the University of Washington and the Bureau of Public Roads (3e). Tests have been made on a number of girder-stiffened bridges in Norway (26) and more tests are planned. It may be hoped that the results of these and a few additional tests can be correlated with Friedrich Bleich's theoretical studies to give the needed information.

(C) *Atmospheric Damping.*—On the other hand, the atmospheric damping can be reasonably determined by tests on a properly scaled oscillating section model (3e). Its shape must, of course, be accurately scaled because, to a large degree, this determines the air forces. The mass must be to proper scale if the measured δ is to be applicable since mass affects δ and ψ as is seen in Eqs. 14, 17, 19, and 20 in Appendix II. For this particular test there is an advantage in making the mass low in a known ratio because the measured δ will then be high in the same ratio and more easily measured. The correct value can be found by applying the corrective ratio. The tests show that the damping capacity and logarithmic decrement for atmospheric damping increase with amplitude and approach zero at low amplitudes as illustrated in Fig. 8. The damping can be regarded as composed of a small and negligible viscous damping (constant with η) shown by the dashed line (curve 5a, Fig. 8) plus a much greater component due to a force that is proportional to the square of the velocity which makes the logarithmic decrement proportional to η (Eq. (20) in Appendix II). For torsional motion the increase with amplitude is even more pronounced.

It is seen that plotting the logarithmic decrement against amplitude does much to reveal the nature of the damping or exciting forces. Further than that, these curves can be used to compute for all amplitudes the rates of energy transfer to and from the vibrating structure caused by the different damping and exciting forces, and their effects can be combined to determine the development of the vibration without the necessity of determining in detail the actual varying forces or pressures (3e).

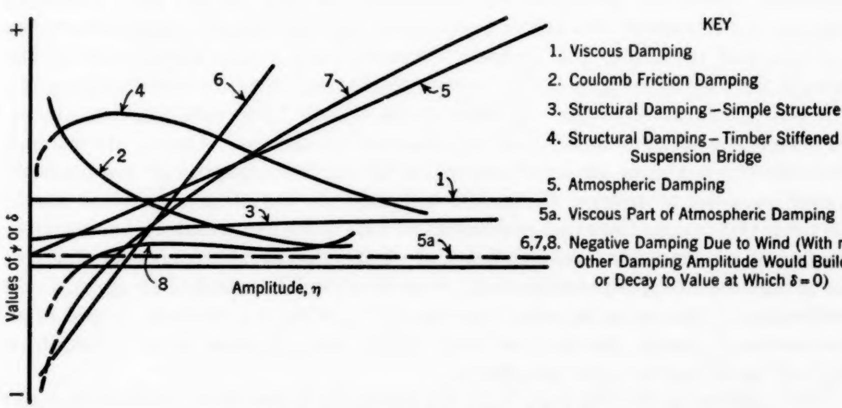

FIG. 8.—TYPICAL CURVES FOR δ OR ψ

11. MODEL TESTING

Much of the present knowledge of the wind-actuated oscillation of suspension bridges has been acquired through model tests and additional testing is recommended in the further pursuit of the problem. It is pertinent, therefore, to state briefly the methods of model testing and the conditions which must be fulfilled if the model indications are to be reliable.

(A) *Static Model.*—A static model must be correctly scaled as to form so that the wind paths, velocities and pressures will correspond to those of the prototype. The model, representing a section of the bridge, must be long enough so the distortion due to the flow of air around the ends will be negligible or it must have "end plates" to prevent such end flow. It is mounted in the wind tunnel with supporting elements attached to scales outside the tunnel in such manner that the vertical and horizontal components of the total force exerted by the wind can be weighed separately and that the moment of the resultant wind force about some convenient axis can be determined. Tests are run at various measured wind velocities and a wide range of angles of attack (3c) (3d) (3e).

(B) *Aerodynamic Model.*—The static model tests supply information concerning wind actuated oscillation of the structure to the extent that the negative slope theory applies but an aerodynamic model capable of performing the essential motion of the prototype is necessary for a full investigation. Whereas force and moment represent the significant wind effects upon a stationary bridge, energy becomes the more important factor in the case of oscillation. Furthermore, time enters the picture in a dominant way and as a result inertia, acceleration, mass, and moment of inertia, as well as elastic and damping forces take on definite significance and must be properly scaled or accounted for. Also such factors as frequency, resonance and phase—entirely absent from the static model tests—become controlling elements. It is true that these factors are subject to treatment on a theoretical basis but this requires additional mathematical steps in the processing of test data and, therefore, increased occasion for deviation between actual and predicted behavior. Moreover, the static tests yield only the steady forces, averaging out, but not revealing, the periodic variation in pressure occasioned by the passage across the section of the vortex discharged from its leading elements. The phase treatment of the variation of the steady force with angle of attack leaves out the effect of these small but resonant variations. On the other hand, if all the properties of an aerodynamic model of the entire bridge are properly scaled from the prototype, the model will move in the same manner as would the prototype under the same conditions and its wave form, distortion, velocity, frequency, acceleration and inertial and elastic forces will represent those of the prototype to appropriate scales. A section model, mounted on springs and oscillating at the same frequency as the full model, is subjected to the same gravitational, elastic, inertial and wind forces that act upon a short length of the full model at the same amplitude.

The studies on the Tacoma Narrows Bridge and also those on the proposed Severn River Bridge in England (22) (23) have been carried on by means of both full and sectional aerodynamic (oscillating) models.

(C) *Scales of Aerodynamic Model.*—It is not difficult to determine the scales of all of the controlling elements of the aerodynamic model. These are fixed by simple laws of geometry and mechanics and three governing conditions which must be satisfied. These conditions are (3s):

(a) The wind forces are inertial; the viscous forces are not significant and can be disregarded (see Art. 1).

(b) The acceleration of gravity, g, acts alike on model and prototype and its scale, $\frac{g_m}{g_p}$, must be unity. (Subscripts, m and p, refer to the model and prototype, respectively. The other symbols are defined in the text.) If the inertial forces are to be proportional to the gravitational forces then the scale of any acceleration, $\frac{a_m}{a_p}$, must likewise be unity.

(c) Natural air acts upon the model and the prototype alike so that the scale of mass density, $\frac{\rho_m}{\rho_p}$, is unity for air and must likewise be unity for all parts so that all dynamic forces will be reproduced to proper scale.

Simple geometry dictates the scales of length, area, volume, and area moment of inertia as n, n^2, n^3, and n^4 respectively, n being the ratio of a length on the model to the corresponding length on the prototype.

With the scale of acceleration fixed as unity it is possible to determine the scale of time from the simple relation, $a = L/t^2$, L being distance and t being time. In other words:

$$t = \sqrt{\frac{L}{a}} \qquad (6)$$

The scale of time can be expressed:

$$\frac{t_m}{t_p} = \sqrt{\frac{L_m/a_m}{L_p/a_p}} = \sqrt{\frac{L_m}{L_p}} = \sqrt{n} \qquad (7a)$$

—remembering that $a_m = a_p$. The scale of velocity follows from those of distance and time:

$$\frac{V_m}{V_p} = \frac{L_m/t_m}{L_p/t_p} = \frac{L_m\,t_p}{L_p\,t_m} = \frac{n}{\sqrt{n}} = \sqrt{n} \qquad (7b)$$

If the scale of mass density is unity then the scale of mass (which is density times volume) must be n^3, the same as the scale of volume. Also weight, $m\,g$, will have the scale, n^3, since the scale of g is 1. Furthermore the inertial forces, $m\,a$, will have the same scale, n^3, since a also is the same on model and prototype. Naturally, all other forces acting on or within the structure must be proportional to the weight and inertial forces and must have the same scale, n^3.

The scale of unit stress is the scale of $\frac{\text{force}}{\text{area}}\left(\text{that is, } \frac{n^3}{n^2} = n\right)$ which is therefore also the scale of modulus of elasticity, E.

Any of the scales can be modified by any convenient ratio provided compensating adjustments of other scales can be made without violating the proper scales of any essential properties. For example, the mass density scale, $n = 1$, can be disregarded for a given truss member or a concrete floor slab provided the outside dimensions are kept true to the linear scale, n, and its weight is kept to the force scale, n^3, by suitable interior loading or hollowing out or choice of material. It is, of course, necessary that the combined weights be correct to

the scale, n^3, and that the weight distribution be correct so that the mass moment of inertia and radius of gyration will be correct to their scales of n^5 (mass times distance squared) and n, respectively.

It is difficult to find material for the essential structural elements of the model (cable and stiffening girder) which meets the scale requirement $\frac{E_p}{E_m} = n$, and has the proper value for other essential properties such as damping capacity (non-dimensional). This difficulty can be avoided by appropriate scale adjustment as shown by the following analysis.

The modulus of elasticity, E, affects the cable stretch and girder deflection which, being linear dimensions, must have the scale, n. Thus the stretch, $\Delta L = \frac{F\ L}{A\ E}$ has the dimensions $\frac{n^3\ n}{n^2\ n} = n$. If E is made the same on model and prototype the necessary condition can still be filled if the scale of $A\ E$ is maintained as n^3 by applying the scale n^3 to stress-carrying area. This has been done on the model of the Tacoma Narrows Bridge. A piano wire with area reduced by the scale n^3 carries the stress; the correct mass and exposed area are provided by sectional steel sleeves strung on the wire with a snug fit at the middle of each section so that they do not slip or make contact, one with the other (3c).

Similarly, a bending deflection, $\frac{F\ L^3}{C(E\ I)}$, having the scale, $\frac{n^3\ n^3}{n\ n^4} = n$, can be kept to correct scale by scaling E at unity and I at n^5, the distorted scale of I applying only to the moment-resisting section. The equivalent of this is done by introducing properly designed springs in series with truss members, which would otherwise be excessively strong, or by carrying the bending stress in a weak beam which is masked by the normally scaled beam, the latter being cut to relieve it of stress (3c) (3d).

While such details as girder flanges do not appear to affect the Strouhal number (Art. 1B) model tests show that they do affect the lift, drag and moment and the oscillation; it is not permissable to omit them on the model. Stringers under the deck of an otherwise fully symmetrical section cause a very noticeable difference in behavior in upward or downward angled winds of the same degree. Curbs may have a similar effect. All edges should be true and their corners should be sharp unless distinctly rounded on the prototype. Small mesh openings as in gratings cannot be scaled down because they then block the wind as does a veil and give the effect of solid panels on the model. (This is an effect of viscosity.) Such a grating can be represented by a grid of coarser mesh than a truly scaled one but with the same solidity ratio. The model should be carefully detailed to keep damping to a minimum to make sure it will reveal any possible behavior of the prototype.

The form of a section model must be built to correct scale, and the mass radius of gyration must be correct. The mass can be reduced in order to get more sensitive readings of the damping as noted in Art. 10C. The model must be rigid so that it will not be appreciably distorted during testing. It is supported by springs designed to permit both vertical and torsional motion or to confine the motion to either type. For investigating flutter the frequencies

of the vertical and torsional components can be adjusted separately as needed (3c) (3e). The aspect ratio, length over width, should preferably be not less than 4.5 or 5.0 or end plates should be used to prevent distorted results due to the flow of air over the ends.

(D) *Choice of Model Type.*—An aerodynamic section model will reveal the velocity at which a catastrophic oscillation of any type will occur under damping conditions similar to those applicable to the model. This is all the information needed concerning such motion. In most oscillations the air forces and the motion of the structure both bear a constant relation to the amplitude so that all forces acting on a unit length of the section model are identical to those acting on a unit length of a full model moving at the same frequency and amplitude. For such oscillations the behavior of the prototype can be calculated from section model tests. There are phenomena, however, which require full models if they are to be investigated only by model tests. Examples are:

(a) The possible effect of gusts of wind blowing nearly parallel to the bridge. Tests at the National Physical Laborabory of the Department of Scientific and Industrial Research of Great Britain on a full model of an early design for the proposed Severn Bridge failed to reveal any synchronized oscillation set up by periodic gusts but did show irregular motions caused by random gusts.

(b) A full model is preferable for investigating or verifying the effects of stays of all types. However, the effect of a simple system of stays can be quite well evaluated by means of section model tests together with the analysis developed by Friedrich Bleich for predicting the effect of stays on frequency and wave form (1m).

(c) The possibility of flutter, coupling any pair of the calculated vertical and torsional modes of the proposed structure can be investigated with a section model designed for the correct vertical and torsional frequencies; however, if the coupled modes have not the same wave form, then their relative amplitudes and phase will not be constant along the span. A full model is best for determining the amplitude of a restricted oscillation of this type. However, it would appear that a fair approximation of it could be computed from the decrement curves for a section model by applying a correction based on Mr. Bleich's factor, D using it (11b) in accordance with its influence on the aerodynamic damping terms, positive or negative (11c). It is improbable that two modes of markedly different form can combine to produce a flutter of serious vigor even though their frequencies may be nearly equal. In any case the tendency to flutter in coupled modes of different form can be no worse than indicated by the section model.

(E) *Instrumentation.*—There are several methods of measuring satisfactorily the wind velocity. For this type of model testing they must be suitable for low to moderate velocities. Pitot tubes are satisfactory for the higher velocities used. Instruments which use a weak calibrated spring to balance the pressure over a rather large opening or those based on the change in electrical resistance of an electrically heated fine wire when cooled by the wind

stream are satisfactory for velocities below the sensitivity of the manometers used with the pitot tubes (3c). The vertical angle of the wind can be measured by noting the angle of a smoke stream or silk fiber in the jet or by means of a "pitchmeter," a horizontal cylinder having two holes about 84° apart in its cylindrical surfaces, these being connected to the opposite sides of a manometer. A graduated scale measures the pitch angle, the angle to which the cylinder must be rotated in order to balance the manometer (3c). The model motions can be picked up by means of small accelerometers or by devices utilizing bonded or unbonded wire resistance strain gages, and recorded on an oscillograph, or by other means (3c).

12. Lessons from Bridge Performance

The investigation of the aerodynamic stability of suspension bridges and of means for achieving satisfactory designs would be incomplete without a study of the records of actual bridges. In the nineteenth century a number of relatively light suspension bridges in Europe and the United States were destroyed or severely damaged by wind action. These were either unstiffened, except for continuity of the floor system, or stiffened by heavy railings or light trusses. Almost invariably the record describes the wind as of hurricane intensity (3q). After an interval of some 80 yr the twentieth century has recorded the collapse of the Tacoma Narrows Bridge, the damage and seriously threatened safety of the Deer Isle Bridge as originally built, and moderate oscillation of the Bronx-Whitestone, Fykesund (Norway) and Thousand Islands Bridges (3a) (20) (21). During 1950 a 580-ft girder stiffened bridge at Beauharnois, Quebec, showed considerable activity. In addition to the foregoing girder-stiffened bridges moderately severe oscillations have been observed on certain truss-stiffened bridges (Golden Gate and the Peace and Liard River bridges on the Alaska Highway). Data concerning these bridges are given in Table I and more information concerning some of them will be found in Appendix III.

Within the scope of the observations the behavior of actual bridges tends to verify the indications of model tests and of theory on three important points.

(1) The motions of the girder-stiffened bridges are either purely vertical or purely torsional, whereas those of the truss-stiffened bridges or bridges stiffened only by the deck structure are coupled vertical and torsional motions (see Art. 4(D)).

(2) Destructive oscillation of truss-stiffened bridges occurs only at very high velocities, whereas such oscillations can occur on girder-stiffened bridges at quite moderate velocities (42 mi per hr on the original Tacoma Narrows Bridge).

(3) Moderate oscillations at low wind velocities have been observed on some girder-stiffened bridges, whereas higher velocities are needed to cause moderate oscillation of truss-stiffened or unstiffened bridges.

The use of diagonal stays or an increase in vertical stiffness has reduced the amplitudes of the oscillations of a number of bridges (for example, stays on

TABLE 1.—Modern Suspension Bridges Subject to Aerodynamic Oscillation

(a) Description and Identification, with Remarks

Item	Bridge	Remarks	Reference[a]
1	Tacoma Narrows, Tacoma, Wash.	Original structure: Several vertical modes at low to moderate velocity; catastrophic torsion above 30 mi per hr; destroyed December 7, 1940.	(3a)(3b)(3c)
2	Thousand Islands, St. Lawrence River, New York to Ontario, Canada	Before stays were added: 24-in. maximum double amplitude in the fundamental vertical mode; with center ties added, the maximum was 15 in. at the main-span quarter point and 12 in. at the center of the side span in one node vertical mode.	(18b)
3	Deer Isle, Me.	Early movements of 6 in. to 10 in. reported in the first asymmetric vertical mode; after the initial installation of stays, 12.5 ft single amplitude in the first symmetric vertical mode developed in a wind of 72 mi per hr on December 2, 1942; a more complete stay system was installed later.	(10c)(19)
4	Bronx Whitestone, East River, New York, N. Y.	Original condition: First asymmetric vertical mode only; maximum observed double amplitude 30 in., more or less; the greatest activity occurred in a quartering wind.	Appendix III
5	Fykesund, Norway	Vertical oscillation with one, two and three nodes; maximum double amplitude, 5.25 ft; no torsion; wind not measured; no movement was reported since stays were installed in 1945.	Appendix III
6	Beauharnois, Quebec	Symmetrical vertical oscillation in a quartering wind across the lake; maximum double amplitude, 13 in.; no oscillation in a normal wind (possibly turbulent); no asymmetric mode.	Appendix III
7	Golden Gate, San Francisco, Calif.	Symmetrical torsional oscillation in winds stronger than about 30 mi per hr normal to the bridge; maximum double amplitude, 45 in.; damped by downward wind approaching the north side span over a hill; first asymmetric mixed torsional and vertical modes, single amplitudes 4 ft to 6 ft on Dec. 1, 1951, in wind reaching 70 mi per hr.	Appendix III (2)
8	George Washington, Hudson River, New York, N. Y.	Original condition: Mild vertical and slight torsional movements.	Appendix III
9	Lions Gates, Vancouver, B. C., Can.	About 3 in. amplitude in the first symmetric mode, in wind estimated at 50 to 60 mi per hr; the heaviest in 15 yr.	Appendix III
10	Peace River, Alaska Highway, B. C., Can.	Alaska Highway: Small motion in a moderate wind; symmetric torsion, estimated 10° double amplitude in a wind of 40 to 50 mi per hr.	
11	Liard River, Alaska Highway, B. C., Can.	With temporary timber deck: Small motion in a moderate wind; torsional motion enough to cause noticeable relative motion between adjacent longitudinal floor planks in a wind estimated at 35 to 40 mi per hr.	Appendix III

(b) Dimensions and Characteristics

Item[b]	Span (Ft)		Sag (ft)	Cables[c] (ft)	Weight (lb per ft)	Stiffening Member			Stiffness Index[d]
	Main	Side				Type	Depth (ft)	Total I (in.^{2}–ft^2)	
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*
1	2800	1100	232	39	5,700	plate girder	8	2,566	160
2	800	350	...	30.5	3,200	plate girder	6	...	350
3	1080	484	108	23.5	2,400	plate girder	6.54	828	190
4	2300	735	200	74	10,940	plate girder	11	5,300	400
5	755	...[e]	95.5	23.6	3,100	I-beam	17.7[f]	36[g]	230
6	580	...[e]	58	30	3,430	plate girder	7.5	1,800	2400
7	4200	1125	470	90	21,300	truss	25	88,000	347
8	3500	610[h]	316	106	31,590	none	...	...	600
9	1550	614	150	40	4,600	truss	15	10,500	400
10	930	465	93	30	4,500	truss	13	10,400	1560
11	542.8	232.7	54.28	30	3,040	truss	8	2,000	2700

[a] See Appendix III, or the reference cited by number in Appendix I. [b] For name and further description, see Table 1(a). [c] Distance, center to center, in feet. [d] "Stiffness Index" developed by O. H. Ammann (not published) is: $8.2\ w/f + 2(0.07\ I/l^4)$ multiplied by a factor for side-span effect, ranging from 0.95 to 0.67. In this equation: w denotes dead weight per foot of bridge, in pounds; f is the sag of the main span, in feet; I is the moment of inertia of both trusses, in in.^{2}–ft^2; and l is the length of the main span in units of 1000 ft. [e] Side spans not loaded. [f] This depth is in inches. [g] Includes the equivalent in steel of longitudinally reinforced concrete deck. [h] Side spans 610 ft and 650 ft.

Deer Isle, Thousand Islands, Bronx-Whitestone bridges; stiffening on Menai Straits, and Bronx-Whitestone bridges). The behavior of stays is in general agreement with the indications of tests and theory (that is they can inhibit or modify certain modes of motion which may occur on a given bridge). They have undesirable features (see Art. 3 and Art. 15) and are generally recommended more for remedial than for original design.

It was natural that the failure of the Tacoma Narrows Bridge should have been attributed initially to the rather extreme flexibility of the structure. Several investigators pursuing studies along this line found a correlation between the stiffness and reported behavior of a number of suspension bridges. Mr. Steinman (10d) used the basic stiffness terms $\frac{H}{l^2}$ and $\frac{E\,I}{l^4}$ multiplied by the weight per foot and divided by the square of the width of the suspended structure. Louis Balog (10e) made different use of the same basic stiffness terms plotted against the main span length, l. Shortridge Hardesty, M. ASCE, plotted the deflection under a half span load of 200 lb per ft against the length of the main span (not published). O. H. Ammann, M. ASCE, computed a "stiffness index," the half-span load per foot necessary to cause a deflection of one foot at the quarter point of the main span (not published). Each of these studies as applied to existing bridges showed a rather consistent relation between the stiffness and the tendency to oscillate. However, the model studies and theoretical analyses made to date have failed to indicate that increased stiffness is an important factor in preventing oscillation, and therefore afford little information for the evaluation of the proposed indices. Obviously, indices of such nature, if they could be firmly established, and their scope and limitations determined, would be of great value to designers.

One modern truss-stiffened bridge sometimes regarded as questionable—the Golden Gate Bridge—is among the more flexible bridges of this type. It is quite exposed to wind from the sea. In fact, it would probably oscillate a great deal more if the uniformity of the west wind approaching it were not disturbed by the hill shielding the north span.

The truss-stiffened suspension bridges over the Peace River and the Liard River on the Alaska Highway exhibit small oscillations in moderate winds up to 30 mi per hr. However, rather large motions have been observed on each during infrequent strong winds. The "stiffness index" proposed by Mr. Ammann exceeds 1500 and 2500 for the Peace River and Liard River bridges, respectively. These two bridges have open-strand cable construction. The vibration of individual strands between cable clamps (27) should not be confused with the oscillation of the bridges as a whole.

The Beauharnois Bridge provides a recent addition to this record of data on actual bridge behavior. It shows that an extremely rigid girder-stiffened bridge can be oscillated if the required wind conditions occur. This particular section with a $\frac{d}{b}$-ratio of 0.25 and one outside sidewalk has not been investigated by dynamic section model tests. However, if its behavior may be estimated from such tests on models with $\frac{d}{b}$-ratios of 0.24 and 0.2875 the bridge

might be expected to show restricted vertical oscillation in very moderate winds and a possibility of severe vertical and torsional oscillation at high velocities. The bridge has remained stable in quite high normal winds, yet oscillates (in the first symmetric mode) in moderate quartering winds. From the model tests it appears that the oscillation could be caused by the normal component of the quartering wind which approaches across Lake St. Louis, and may be uniform and quite steady. The normal wind of high velocity may be turbulent or may be deflected by the local topography and building (also there may be a question about its actual velocity and direction; see Appendix III). Longitudinal components of gust action, referred to in connection with the Bronx-Whitestone Bridge, cannot be considered as an explanation because the asymmetric mode which such action can excite has not been observed here. The behavior of this bridge is discussed in some detail in Appendix III.

The behavior of this stiff bridge supports, in a striking way, the indications of tests on the Tacoma Narrows and Severn Bridge Models as cited in Paragraph (c), Art. 18 (B); that is, that increased vertical stiffness alone is not always to be relied upon to eliminate a seriously objectionable oscillation.

Of the three relatively stiff short-span bridges listed in Table I the girder-stiffened Beauharnois Bridge has shown objectionable motion, while the two truss-stiffened bridges on the Alaska Highway have indicated the possibility that they may develop objectionable motion. On the other hand the truss-stiffened Lion's Gate Bridge of relatively low stiffness has shown negligible motion under severe exposure to wind. Some ten other bridges of relatively low to moderate stiffness not listed in Table I (Mount Hope in Rhode Island, St. John across the Willamette river at St. Johns, Ore., Ambassador at Detroit, Mich., Mid-Hudson at Poughkeepsie, N. Y., Ile d'Orleans across the St. Lawrence river at Quebec, Can., Transbay between San Francisco and Oakland, Calif., Brooklyn and Triborough over the East river at New York, and several Scandinavian bridges of relatively short span) are not known to have experienced motions under wind action. While no records of wind intensity and direction at the sites of these bridges are available, it must be assumed that most of them at times have been subjected to severe wind action during the 15 yr to 68 yr of their lives.

Table I and Appendix III describe only those modern bridges which are known to have oscillated under wind action. The recorded behavior of a much larger number of suspension bridges of a wide variety of structural characteristics and range of span length leads to the conclusion that only a very small proportion of suspension bridges have oscillated significantly due to wind action, although many of them must have been subjected to winds of high velocity.

Most of the modern girder-stiffened bridges and the narrow, essentially unstiffened early bridges built principally in England and the United States, are characterized by low stiffness. Among these bridges a relatively large porportion have experienced oscillations under wind action. Of the stiffer girder-stiffened structures the Beauharnois Bridge has developed oscillations, whereas the Toledo Bridge has not. The long-span extremely heavy and unstiffened George Washington Bridge has experienced insignificant motions.

The interpretation of the history of suspension bridge behavior is necessarily dependent upon what other knowledge is taken into account and opinions differ with the weight given to various findings. Statistically, the record may be interpreted as evidence that stiffness has a determining influence on the stability of suspension bridges against motion under wind action and some regard stiffness as the dominant factor. Others do not accept this appraisal; first, because model tests show a marked susceptibility to oscillation of the girder-type sections regardless of stiffness and, second, because rational analysis indicates only moderate benefits from stiffness unless it may give rise to a considerable increase in damping. More information is needed before these views can be reconciled.

There is general agreement that natural winds usually lack the degree of uniformity and steadiness required to build up oscillation and that some bridges which do not move might do so at other sites where the wind is steadier and more uniform.

Nonuniformity of wind action over the length of the structure is likely to be of relatively greater influence on long spans than on short ones. This may account for the susceptibility to motion of the relatively stiff short-span bridges listed in Table I, while none of the large number of stiff bridges of medium and long span have experienced motion.

Very little has so far been observed and recorded on the characteristics of the wind acting on existing suspension bridges.

II.—DESIGN CRITERIA AND METHODS

13. The Problem

Engineering experience, in general, has developed many problems of vibration and their solutions. Objectionable vibration must be dealt with in almost every machine or dynamic mechanism. The solutions are found in three general methods of attack; redesign to (a) weaken or eliminate the source of vibration, (b) break up resonance, and (c) introduce damping. Road surfaces and tires are improved to reduce the shocks applied to an automobile; the springs, wheel-base and weight distribution are manipulated to minimize the objectionable resonance of the car with the unavoidable road shocks and, finally, shock absorbers remove energy built up in vibration and thus limit the amplitude and the disagreeable acceleration felt by the passengers.

The problem of suspension bridge oscillation is not unique but resembles other vibration problems and can be solved by (1) fundamental changes in form which greatly modify the wind action causing the oscillation, (2) altering the frequency of the wind force by less drastic changes in form or altering the natural frequency of the bridge by changes in stiffness, in order to avoid resonance of the exciting force with the bridge motion and (3) damping out of the remaining, less potent vibration.

14. Means Available to the Designer

The latitude within which the designer can act to prevent objectionable oscillation is determined by the degree to which he can control the physical

properties of the bridge. His control of various properties is limited by the physical characteristics of materials, the live loading and also by economics, as follows:

(a) Span length, side span-main span ratio, and sag ratio can be manipulated within rather narrow limits without disproportionate increase in cost.

(b) Dead load mass can be reduced in preparing the design specifications by sacrificing load capacity or discounting future development and can be controlled within restricted limits without such sacrifice through some choice of materials of differing weight, efficiency and cost. It can be increased materially only at corresponding increase in cost.

(c) Stiffness can be increased by reducing the sag or increasing the moment of inertia. The former may range from, say 1/9 to 1/12 of the span. Moment of inertia can be greatly varied by using girders or trusses, which provides a considerable range of overall stiffness on spans under 1000 ft but a much smaller practical range for spans over 2000 ft. Torsional stiffness can be greatly increased by using both top and bottom laterals. Towers become increasingly flexible as the span increases and cannot be economically stiffened in bending enough to influence behavior though their torsional resistance can be increased to some extent.

(d) A great range of shapes is possible through the choices of girder or truss stiffening, deck, through or half-through construction, outside or inside sidewalks, solid, grated or slotted decking, etc.

(e) Friction can be considerably increased through design details or special mechanisms.

15. Avoiding Sources of Excitation

(A) *Girder Versus Truss Types of Stiffening Member.*—When a designer chooses between the girder and truss types of stiffening member he determines the type of wind action to which the bridge may be subjected, that is, predominantly vortex action (and possibly "negative slope" action) on the girder section, or flutter on the truss or extremely shallow girder section. In the former case, a tendency to restricted or catastrophic oscillation (torsional or vertical or both) may be expected at moderate velocities in a steady, uniform wind. In the latter, the response, if any, will probably occur at higher velocities but is likely to be catastrophic unless modified by means described below.

(B) *Stability of Girder Sections.*—Although girder sections are subject to oscillation over a considerable range of velocities, it will probably be found that they can be proportioned to avoid serious wind action within the expected velocity range. Certainly the $\frac{d}{b}$-ratio has a strong influence. Mr. Steinman and F. J. Maher have predicted their behavior from static model tests and indicate certain ranges of the $\frac{d}{b}$-ratio in which catastrophic vertical or torsional motion is not expected to occur (18). These indications should not be fully relied upon until verified by dynamic section model tests. (One such verification has been made at the University of Washington where it was confirmed

that the vertical oscillation is catastrophic for d/b greater than 0.24 but is restricted for smaller $\frac{d}{b}$-values (3c).

The behavior of air flow around various types of sections suggests that girder-stiffened bridges might be stabilized by outside sidewalks or fins. Messrs. Steinman and Maher found this to be critical in static model tests and it appears that only certain ratios of fin width to girder depth may be favorable. A limited series of tests, on dynamic section models at the University of Washington showed little or no benefit from the fins. In fact they were unfavorable with one deck combination (3c).

It may be expected that roadway or sidewalk openings can be arranged to improve the stability of girder sections; however, the University of Washington tests showed definitely that girder sections with open roadway or open sidewalks or even a completely open deck gave torsional activity at a lower wind velocity than was found with the solid deck (3t).

(C) *Stability of Truss Sections.*—Many details of truss-stiffened sections have only a secondary damping effect in the wind although leading edge details near the plane of the deck have more positive influence. On the other hand, a system of slots definitely changed a catastrophic flutter of a proposed Tacoma Narrows design (with single lateral system) to a weak restricted oscillation easily damped out (3d). It does not follow that the identical system will be equally effective on a truss section of distinctly different form. It is quite likely that a completely open deck (that is grating) would weaken or prevent flutter of a truss-stiffened section. A beneficial effect which is dependent upon special minor details which may be altered within the life of the bridge should not be relied upon.

16. Avoiding Resonance

(A) *Alteration of Wind Action.*—With a knowledge of the $\frac{V}{N\,b}$-ratio obtained through section model tests and of the natural modes of motion of the bridge calculated from the mass and elastic properties of the proposed design, a study can be made to determine whether or not the critical velocity of an objectionable oscillation which may be present can be pushed up beyond probable occurrence. In general, high frequency is an advantage but in many cases the increased frequency obtained by even costly stiffening will not be adequate to avoid objectionable oscillation. In such cases the other methods must be relied upon such as altering the wind action through a major change in shape or (if the motion is not catastrophic) increasing the damping.

(B) *Adjustment of Frequency.*—In the case of flutter, frequency adjustment can be readily used to throw the vertical and torsional components out of resonance (1j) (11). The use of top and bottom laterals can accomplish this in almost any case. It appears that if the torsional frequency is $2\frac{1}{2}$ or possibly 2 times that of the corresponding vertical mode, flutter is highly improbable within any likely wind range (3e). If the stiffening system is too shallow to make effective use of top and bottom laterals, the torsional frequency can be increased by using several cross frames made up of struts between the cables

with diagonal stays from each cable to the opposite girder, provided these can be located away from the node points of the torsional modes which must be modified (3d). (This treatment is undesirable in original design but may be used to improve an existing bridge (21).)

(C) *Center Ties and Diagonal Tower Stays.*—Center ties and diagonal tower stays can be used to break up resonance in the sense that they tend to inhibit certain modes (3a) (3c) (3d). Center ties, for example, effectively prevent the first asymmetric torsional mode (unless the towers are very weak in torsion). Diagonal stays, if attached to the truss near points of maximum amplitude inhibit the early development of a mode though single sets of stays may not always prevent its ultimate development if the excitation is vigorous (3d) (19).

In using ties and stays, certain design problems must be faced. Unyielding center ties restrict the normal deformation of the main span under certain possible though improbable asymmetric live loadings. They and the lateral truss must be made strong enough to sustain the resulting stresses (a difficult thing) or, preferably, relieving devices should be introduced so that they may gradually relax as the loading condition develops. Center ties with hydraulic relieving links are used on the new Tacoma Narrows Bridge. They yield under heavy slowly applied traffic loads but resist the briefly applied longitudinal forces which result from oscillatory wind action.

Diagonal stays must be designed for the static loading to which they will be subjected and their effects throughout the design temperature range should be considered. For example, stays from the tower upward toward the cable near the quarter-point act against the main cables and therefore a drop in temperature doubly increases their tension. Stays must have considerable initial tension in order to be effective which requires adjustment of the hanger loads at adjacent panels to prevent distortion of the gradeline.

17. Increasing Damping Capacity

Model tests have shown that a catastrophic oscillation may be reduced to a mild restricted form by modifying the wind forces through alteration of the shape and the flow pattern or by control of resonance by increasing torsional stiffness without changing vertical stiffness. The milder oscillation would probably not damage the structure but might be objectionable from a psychological standpoint. In such a case, a moderate increase in the structural damping is sufficient to reduce the oscillation to negligible proportions. For example, it was found in full model tests of the tentative design for the Tacoma Narrows Bridge with one lateral system that the restricted torsional oscillation which remained after slots were provided in the deck to eliminate the catastrophic flutter could be completely prevented (3d) by damping having a total logarithmic decrement of 0.05.

Mr. Bleich has shown (1i), as previously discussed, how the normal damping of a bridge can be increased, possibly doubled, at low amplitudes by effectively designing to develop friction. Methods suggested are:

(a) Designing all stringers for bearing on the floorbeam, using an extended spacing of deck expansion joints and bracing the floorbeams laterally so that

the relative movement will cause sliding instead of horizontal deflection of the floorbeam.

(b) Placing the deck near the plane of the top or bottom chord and using stiff connections so that the floor slab participates in the chord strains and thereby stores energy in the slab where the structural damping is presumably higher than in the truss.

(c) Providing end friction. This is done in the design of the new Tacoma Narrows bridge by introducing hydraulic cylinders between the truss and the tower with the apertures designed for slow release of pressure. Similar units are provided in the center cable ties (24). Such units give a considerable damping effect in addition to their function of slowly yielding to gradual stress changes. Mechanical friction "tongues" were used at the towers of the Bronx-Whitestone bridge to brake longitudinal motion of the suspended structure (19).

(d) If the structural damping of the truss or girder is found to be greater than that of the cable (not clearly established at the present time) the overall damping can be increased by increasing the proportion of the total energy that is stored in the stiffening member. This can be accomplished to some extent by increasing the cable sag and the moment of inertia of the stiffening member.

18. Effect of Weight and Stiffness

In the foregoing application of the three primary methods of preventing vibration no direct reference is made to weight and vertical stiffness for the reason that their influence is complex and requires special consideration.

(A) *Weight.*—Weight influences the wind forces and the possibility of their resonance with bridge motion only through its effect on the motion. If all of the stiffness is due to the loaded cable an increase in weight has no effect upon the frequency. With a substantial girder or truss increased weight reduces the frequency, usually an unfavorable effect (see the last paragraphs of Art. 2 and paragraph F of Art. 3).

An increase in weight without reduction in frequency tends to reduce the structural damping by increasing m in Eq. 17 and 19 in Appendix II. There will also be a tendency for the increased stress or cross sectional area to increase the constants in these equations in partial compensation for the increase in m but this will not affect damping due to truss action which is not stressed by dead load, only by deflection. On the other hand, if the truss stiffness is sufficient to influence the frequency an increase in weight will reduce ω^2 in Eqs. 17 and 19 which also tends to offset the increase in m. Summing up, an increase in weight cannot be expected to make a significant change in structural damping.

The greatest influence of an increase in weight lies in the fact that it increases the energy storage in the same ratio and thereby decreases by the square root of that ratio the amplitude which can be built by a given input of energy from an outside source. It also increases the time required for a given rate of energy input to build up a given amplitude.

(B) *Vertical Stiffness.*—An increase in vertical stiffness in a certain ratio increases the frequency by the square root of that ratio except when the in-

creased stiffness results from increased weight which increases both spring constant and mass and so has only a slight secondary effect upon frequency (as noted in the preceding paragraph and in Arts. 2 and 3).

For longer spans the degree to which the frequency may be increased by increasing the vertical stiffness is quite limited but there are other benefits derived from stiffness as follows:

(a) Model tests and some field data suggest that sizeable amplitudes in the first asymmetric vertical mode can be caused by the action of eddies or random gusts of wind acting nearly parallel to the bridge axis. The oscillation in this case does not arise from the gradual input of energy but develops as a result of the momentary displacement when the bridge yields to the direct pressure of the gust (a force several times the magnitude of the alternating components of pressure which, due to resonance, gradually build up motion in a cross wind). The initial displacement under this direct wind pressure is inversely proportional to the first power of the stiffness (in contrast to a frequency increase proportional to the square root of the ratio of stiffness increase). Thus an increase in stiffness which can cause only a moderate increase in frequency may be sufficient to inhibit oscillation resulting from longitudinal or quartering gusts (see "1. Bronx-Whitestone Bridge," Appendix III).

(b) The energy storage at a given amplitude increases in direct proportion with the overall stiffness and the limiting amplitude for a given energy input is decreased as the square root of the ratio of increase in stiffness.

(c) An increase in vertical stiffness brought about by increased weight increases the denominator of Eq. 20, which interprets the general character of negative aerodynamic damping, and so reduces ψ and δ. Using this reduced value of δ in Eq. 12b necessarily increases the time, t, required to bring about a given change in the amplitude, η, since weight generally has only a small effect upon frequency (N,ω).

An increase in truss stiffness, on the other hand, if effective at all, increases the frequency, ω, and does not change ψ and δ in Eq. 20. Therefore if δ is unchanged and ω is increased in Eq. 12b the time required will be reduced. Actually such an effect of increased truss stiffness may not occur because the negative atmospheric damping in a wind stream is not quite as simple as expressed by Eq. 20 for the positive damping. For example, its variation with the amplitude, η, is more complicated as shown in Fig. 8.

Any factor which increases the time required for a motion to build up in the wind may have a beneficial effect much greater than is suggested by the percentage increase because of the sharply reduced probability of the wind continuing within the required range of conditions (velocity, direction, and uniformity) for the additional length of time.

This benefit will vary in individual cases depending upon the wind required to excite motion of the section. For example, each vertical mode of the original Tacoma Narrows Bridge was excited only in a limited range of velocity; if the velocity changed beyond this range the mode was damped out and a different mode might begin in its place. The field records show this continuous changing of modes without serious amplitudes being built up in any one. On the other

hand, after the center stays were disabled the torsional mode by which the bridge was destroyed could be built up continuously at any velocity above 20 mi per hr (3a) (3c).

The indication of model tests is that increased vertical stiffness alone is not always to be relied upon to eliminate a seriously objectionable oscillation. Tests were made on the full model of the new Tacoma Narrows Bridge with solid deck and one lateral system but with the truss replaced by a 12-ft plate girder to give the $\frac{d}{b}$-ratio, 0.2, of the original bridge (3e). However, the stiffness of the basic model skeleton was excessive for any girder, being equivalent to that of 12-ft girders having 200 sq in. of metal in each flange and more than ten times the stiffness of the girders of the original bridge. Notwithstanding this additional stiffness the model showed the general behavior of the original bridge and its full model. Also despite the stiffness of the 25-ft trusses of an early proposed design of the Severn Bridge, the full model thereof when masked by paper to give the general shape of a girder section, showed the typical behavior of the original Tacoma Narrows Bridge.

The same point is demonstrated by a bridge built in 1949 at Beauharnois, Quebec, with a span of only 580 ft, unloaded back stays and stiffening girders 7.5 ft deep which, though very stiff, has oscillated with a double amplitude of 13 in. in a moderate wind.

19. Design Procedure

At the present time the aerodynamic phase of the design of a suspension bridge must be accepted as analogous to the static analysis of an indeterminate structure. The individual effects of various factors of shape, mass and stiffness, are known in a general way and may be incorporated in the design but it is necessary to investigate the overall design to determine what will be the actual behavior of the structure. Just as there are methods for analyzing an indeterminate structure to determine the stresses after the sections have been assumed, so methods have been developed for predicting with more or less reliability the aerodynamic behavior of a suspension bridge after its various design factors have been selected.

20. Predicting Probable Behavior

(A) *Preliminary Investigation for Flutter.*—If the structure is stiffened by a truss or by an extremely shallow girder and therefore has the characteristics of a flat plate, a very good indication of its probable behavior can be obtained by applying the flutter analysis developed by Mr. Bleich (1) (11). If this indicates flutter within the possible wind velocity range, one can be quite positive that a drastic change is needed. Moreover, it can be expected that, if flutter is indicated by the calculations, restricted oscillations of undesirable amplitude may be expected at a somewhat lower velocity.

A dynamic section model should be constructed and tested in a wind tunnel. This will supply data for use in Mr. Bleich's analysis to determine the modifying

effect of vortex action (11d). A few further tests will provide directly the $\frac{V}{N\,b}$-ratio of the resulting oscillation in the ideal horizontal wind and in vertically angled winds of any degree which might be expected (3e). If the section model shows a catastrophic motion it is only necessary to note its critical velocity (prototype) and decide whether the wind at the site can reach that velocity.

(B) *Prediction of Severity of Restricted Oscillation.*—A moderate extension of the section model tests on either girder-stiffened or truss-stiffened sections will supply information from which the probable severity of restricted oscillations can be determined. This extended technique consists simply of testing the model at various velocities and recording the rate of increase or decay of oscillation, from which the logarithmic decrement can be computed and plotted against the amplitude. Associated with these tests are similar determinations in still air from which the aerodynamic damping of the model (and of the prototype as well) and the structural damping of the model set-up can be measured separately.

The frequency and wave forms of any probable modes can be readily calculated from the geometric, elastic, and mass properties of the proposed design and the energy of vibration of any mode can likewise be calculated. Knowing the rate of energy transfer per unit length of bridge at any amplitude (from the section model tests and the curves for logarithmic decrement) the rate of energy transfer to the prototype can be calculated by a single process of integration for the known variation of amplitude over the length of the structure (3e).

It is estimated that the probable amplitude of a restricted oscillation of the proposed bridge can be determined by this procedure to about the same degree of precision that applies to the calculation of stress in many phases of conventional design provided a reasonably accurate figure for the structural damping of the prototype is available.

21. Wind Exposure Probability

Knowledge concerning the wind conditions upon which the aerodynamic solution must be predicated is comparable to the uncertainty concerning stream flow conditions for which a bridge must be designed. Tests show that a wind of steady velocity uniformly distributed over the bridge will be more severe than a variable or non-uniformly distributed wind. Also for each design there is a certain angle of attack which causes greatest excitation. Common experience and a few special field observations give some idea as to the degree to which these unfavorable conditions may occur.

No general discount ratio can be assumed but observations at a given site may establish quite definite limitations on the severity of wind conditions which must be provided for. For example, tests on a section model of the Golden Gate bridge showed that vigorous oscillation is to be expected in a normal wind of moderate intensity approaching at an upward angle of a few degrees. Considerable excitation was also indicated in a horizontal wind (3d) (11e). Nevertheless, detailed measurements of the movement of this bridge

over a period of years have shown that much of its motion in the wind is more or less random. Even when the double amplitude reached 20 in. to 45 in. in the dominant symmetric mode, large amplitudes were maintained for only a few cycles in immediate response to a wind gust. The very probable reason for the failure of symmetric modes to build up steadily stands out quite clearly. The 1,000-ft hill just northwest of the bridge causes any normal wind to be deflected sharply before it strikes the north side span. The prevailing westerly wind is turned down perhaps 30° and the rare east wind is turned up correspondingly. (Full model tests of the first design for the new Tacoma Narrows bridge with solid deck showed that a vigorous oscillation set up by a wind stream can be quickly brought to rest by simply turning sharply downward the wind which strikes one side span (3d).) On December 1, 1951, in a wind which reached 69.5 miles per hour at the mid-point of the main span of the Golden Gate Bridge and probably exceeded that velocity elsewhere on the bridge, the motion was primarily a one-node coupled torsional and vertical oscillation which reached double amplitudes of about 12 ft and 8 ft, respectively, at the southeast and southwest quarter-points (2.5° to 12.8° double amplitude in torsion depending on the phase of the coupled modes). This asymmetric motion can occur without participation of the side spans. Hence it was not impeded by the distorted wind action on the side spans to the degree which would be expected for the symmetric modes which require side-span participation. However, there was considerable oscillation of the side spans and the interference may have exerted a moderating influence.

A program of section model tests is under way for determining the behavior of certain existing bridges under laboratory controlled wind conditions. It is expected that the comparison of these indications with the actual observed behavior of the bridges will give a general idea as to the extent to which idealized predictions may be discounted for field conditions.

While considering the difference between actual response and idealized predictions of the behavior of existing bridges it is pertinent to speculate, perhaps, as to what these bridges might do at other, less favorable sites. For example, a bridge similar to the Golden Gate Bridge, if located in flat terrain and subject to hurricane winds, very likely would develop serious oscillation. It is hardly to be expected, furthermore, that of all modern bridges, this is the only one which might show such behavior at a less favorable site.

22. Tolerance

It yet remains to be determined what amplitude of oscillation may be tolerated in a suspension bridge. Due to their flexibility, amplitudes of several feet in the lower modes will have no serious structural effect on the larger suspension bridges but it is probable that any oscillation which becomes noticeable to the public should be considered objectionable. In the case of vertical oscillation this probably hinges largely on the acceleration, which produces the sensation of movement and which in turn, is a function of the span and the amplitude so that the permissible amplitude might be expressed as a rather simple function of the span after the necessary experimental con-

stants have been determined. (However, the visual sensation of the large vertical movements of the original Tacoma Narrows Bridge was more objectionable than the acceleration.) If the bridge is used by pedestrians, a very small amplitude in torsion can be detected by sighting from one sidewalk across the opposite rail.

It was observed on the Bronx-Whitestone Bridge as originally built that double amplitudes appreciably over 12 in. in the first asymmetric vertical mode (frequency about 8.5 cpm) were sufficiently perceptible to cause discomfort and possible panicky feeling in some people (unpublished report by Mr. Ammann). This represents $\frac{1}{1150}$ of the wave length and an acceleration of less than one ft per sec^2.

III.—SUMMARY OF CRITERIA AND ADDITIONAL DATA REQUIRED

23. Criteria

There are some rather definite criteria for designing suspension bridges to avoid objectionable oscillation but the influence of some factors cannot be stated quantitatively at the present time with any degree of precision except after tests of the particular section involved. The criteria may be summarized as follows:

A. *Selection of Stiffening Member.*—The use of a truss instead of a girder for the stiffening member will probably avoid the frequent annoying oscillation in moderate winds such as occurred on the original Tacoma Narrows Bridge although the behavior of the Golder Gate Bridge shows that a truss-stiffened bridge may also be, to a lesser degree, subject to such oscillation.

B. *Truss Sections.*—Flutter of a truss-stiffened section can, in all probability, be avoided by using both top and bottom lateral systems or by a suitable system of openings in the deck. With present knowledge it is advisable to verify their effect on a proposed design by analysis and by wind tunnel tests of a dynamic section model.

C. *Girder Sections.*—It is quite probable that girder stiffened sections can be used safely by using a favorable $\frac{d}{b}$-ratio or some favorable form combination such as outside sidewalks, slots or fairing but at the present time such remedies should be relied upon for a given design only if their value is confirmed by aerodynamic tests of a section model of the particular design. If tests indicate that a moderate increase in frequency is desirable to raise the critical velocity above the probable velocity at the site this may sometimes be accomplished by a practicable increase in stiffness (most effectively through a reduction in sag). Damping can be increased if necessary at moderate cost by attention to ordinary design details and by introducing frictional devices.

D. *Analysis and Model Tests.*—By mathematical analysis and dynamic section model tests the aerodynamic stability of a given design under idealized wind conditions can be adequately determined. Until more specific knowledge

accrues, it is advisable to make such investigations of any important suspension bridge unless it closely resembles a bridge known to be stable under the conditions at the proposed site.

24. Further Research Needed

A. *Character of Natural Winds.*—A very important requirement is to determine the characteristics of natural winds including the constancy of wind velocities and angle of attack and the uniformity of wind across a bridge site. These investigations should indicate the degree of uniformity to be expected from various types of storms in certain regions as well as the modifying influence of topographic and cultural features. Recording instruments showing velocity, wind direction and vertical angle at different locations adjacent to actual bridges should contribute materially to the accumulation of this necessary information. These observations should be closely correlated with the behavior of the bridge. The probable behavior of the bridge in idealized winds should be determined by mathematical analysis and aerodynamic section model tests to provide a basis for comparison to indicate the relative effectiveness of the actual wind.

B. *Structural Damping of Suspension Bridges.*—Data should be obtained as the structural damping to be expected in various types of suspension bridges. Theoretical analysis may go a long way toward indicating the probable values but it is believed tests on at least a number of actual bridges will be necessary. These tests need to cover a reasonable range of distribution of the energy of vibration between the cable and the truss, and should give some indication of the damping with different types of floor system since the latter may very likely provide a large share of the total damping. Usual methods of oscillating a structure by means of eccentric weights, etc., can hardly be used because of the extremely slow vibration of suspension bridges. Some preliminary tests have been made on small bridges. Methods have been studied for making measurements on major bridges (3e).

C. *Catalogue of Characteristics of Bridge Sections.*—A series of section model tests is needed, first to determine in a broad way the effects of various shape factors $\left(\frac{d}{b}\text{-ratio, fins, slots, deck or through construction, etc.}\right)$ and second to build up eventually a catalogue of quantitative data on the aerodynamic characteristics of various bridge sections. Such tests will confirm or modify various theories as to the behavior of sections and in time may establish certain rather broad limitations within which a type of design may be used without detailed tests.

D. *Improvement of Methods for Predicting Behavior.*—Correlation studies of full model and section model tests show discrepancies in critical velocities which can be as much as 20%. Part of this can be attributed to the limited precision of controls and measurements but part may be due to the influence on the $\frac{V}{N\,b}$-ratio of some factors which are not fully understood. Further investigation should clear up this uncertainty and increase the precision and

reliability of the predictions. It may be expected also that the method of making predictions can be simplified.

E. *Effect of Weight and Stiffness.*—There is evidence that weight and stiffness beyond the minimum requirements to meet static conditions may have more beneficial effects than can be rationally explained at this time. The effect of weight and stiffness should be the subject of further investigation.

F. *Camber, Stays and Longitudinal Forces.*—There are a number of subsidiary questions, many of which should be cleared up in connection with the above listed investigations. The use of a full model will facilitate the further investigation of such features as camber, stays, and logitudinal forces although they can be treated analytically also.

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- George D. Whittle, M. ASCE, Division of San Francisco Bay Toll Crossings

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* Deceased. † Retired. ‡Resigned.

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January, 1952

APPENDIX I.—LIST OF REFERENCES

Numbers in parentheses in the text of this report refer to the following reading references:

1. "Mathematical Theory of Vibration in Suspension Bridges," by Friedrich Bleich, C. B. McCullough, Richard Rosecrans and George S. Vincent, Bureau of Public Roads, U. S. Department of Commerce, Government Printing Office: (a) Fig. 19; (b) Art. 8, Chapter 4 and Tables 8, 9 and 10; (c) page 156; (d) Chapter 5, Parts 2 and 3; (e) Fig. 15; (f) Fig. 17 and Tables 2 and 5; (g) Chapter 5, Part I; (h) Fig. 16 and Table 6; (i) Chapter 6; (j) Chapter 7; (k) Eq. 6.20; (l) Fig. 73; (m) Chapter 5, Part 4.
2. "The Failure of the Tacoma Narrows Bridge; A Report to the Federal Works Agency," by O. H. Ammann, Theodore von Kármán and Glenn B.

* Deceased. ‡ Resigned.

Woodruff, March 28, 1941: (a) Fig. 25; (b) Figs. VIII–18 to VIII–21, Appendix VIII; (c) Chapter I, Appendix VI and drawings 4 and 5; and, (d) Chapter IV.

3. "Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge," University of Washington Engineering Experiment Station Bulletin No. 116:
   (a) Part I, "Investigations Prior to October, 1941," by F. B. Farquharson;
   (b) Part II, "Mathematical Analyses," by Frederick C. Smith and George S. Vincent;
   (c) Part III, "The Investigation of Models of the Original Tacoma Narrows Bridge Under the Action of Wind," by F. B. Farquharson, June, 1952;
   (d) Part IV, "The Investigation of Models of the New Tacoma Narrows Bridge Under the Action of Wind," by F. B. Farquharson (in preparation);
   (e) Part V, "Amplitude Predictions, Damping Tests, and General Investigations," by George S. Vincent (in preparation);

   Supplementary text references—(f) see Part I, Chapter VIII; (g) see Part I, Figs. 24 and 26 and Chapter VII; (h) see Part I, Chapter III and Appendix III; (i) See Part I, Figs. 9 and 10; (j) see Part II, Figs. 5, 6, 7 and 9; (k) see Part II, Fig. 8; (l) see Part II, Chapter VI and Appendix V; (m) see Part II, Eq. 2.9 and the expression for μ in Art. 6; (n) see Part II, Eq. 2.8 and the expression for μ in Art. 6; (o) see Part II, Eq. 3.5 and Art. 15; (p) see Part I, tables IX and X; (q) see Part I, Appendix I; (r) see Part I, Chapter IV; (s) see Part I, Chapter V; (t) see Part III, Table V; and, (u) see Part II, Chapter VII.
4. "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill Book Co., New York, Third Ed., 1947: (a) Art. 59; (b) Art. 75; (c) Art. 32; and (d) Art. 13.
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6. "The Frequency of the Eddies Generated by the Motion of Circular Cylinders Through a Fluid," by E. F. Relf and L. F. G. Simmons, Reports and Memoranda No. 917, June 1924, Technical Report of Aeronautical Research Committee for 1924–25 (London).
7. "Problems of Aerodynamic and Hydrodynamic Stability," by D. B. Steinman, Proceedings of the Third Hydraulics Conference, Bulletin 31, University of Iowa Studies in Engineering.
8. "Vibration and Drag of Towing Cables in a Uniform Stream," by L. Landweber. Memorandum, David Taylor Model Basin, Jan. 13, 1945.
9. Note IV, "Mechanism of Fluid Resistance," by Theodore von Karman and H. Rubach, U. S. National Advisory Committee for Aeronautics Technical Reports 24–50, 1918. Government Printing Office (Translation from paper in Physikasische Zeitschriff, January 15, 1912).

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11. "Dynamic Instability of Truss-Stiffened Suspension Bridges Under Wind Action," by Friedrich Bleich, Transactions, ASCE, Vol. 114 (1949), p. 1177 and: (a) Discussion beginning p. 1225; (b) Eq. 45; (c) as indicated in Eqs. 46; (d) Part 6; and, (e) p. 1214.
12. "Aerodynamic Theory of Bridge Oscillations," by D. B. Steinman, Transactions, ASCE, Vol. 115 (1950), p. 1180.
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18. "Wind Tunnel Tests of Suspension Bridge Section Models," by F. J. Maher, D. Frederick and E. R. Estes and by D. B. Steinman, Bulletin No. 69, Engineering Experiment Station Series, Virginia Polytechnic Institute.
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22. "The Severn Bridge Aerodynamic Research," by R. A. Frazer (National Physical Laboratory, London) Nature, Vol. 163, p. 7, Jan. 1, 1949.
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APPENDIX II.—ENERGY AND DAMPING

The total energy of vibration of the bridge is expressed by the equation (to be found in most books on vibration (3u)(4c)):

$$U = \frac{w\,\omega^2}{2\,g}\int_L \eta^2\,dx \qquad (8)$$

in which: U is the total energy of vibration; w is the weight per foot of the structure; g is the acceleration of gravity; ω is the circular frequency (that is, $2\,\pi\,N$); and, η is the amplitude at any point, x, along the span. The integration over the length of the bridge is necessary in arriving at the total because η varies along the span but can be expressed as a function of x.

If the bridge oscillates at a steady state, neither increasing nor decreasing in amplitude the total energy remains constant regardless of the instantaneous position. The total energy, U, is the sum of the instantaneous values of the kinetic energy, T, and the potential energy, V. The kinetic energy, $T = \frac{m\,v^2}{2}$, has the maximum value given by Eq. 8 which occurs at the instant all parts of the bridge pass at maximum velocity through their normal dead-load positions. At that instant the potential energy is zero. At maximum displacement the bridge is momentarily at rest and has no kinetic energy, all of the energy being stored as potential energy due to the position of the mass and the elastic strain in the bridge members. It will be noted that the energy of vibration increases with the square of the amplitude.

Considering, for simplicity, a unit length of the structure having a mass, m, and vibrating at the frequency, ω, and the amplitude η, then (see Eq. 8):

$$U = \frac{m\,\omega^2\,\eta^2}{2} \qquad (9a)$$

If, due to the action of damping or frictional forces, the amplitude is reduced in one cycle to $\eta - \Delta\eta$ then the energy is reduced to

$$U = \frac{m\,\omega^2}{2}\left[\eta^2 - 2\,\eta\,\Delta\eta + (\Delta\eta)^2\right] \qquad (9b)$$

and by subtracting Eq. 9b from Eq. 9a the energy loss is

$$\Delta U = \frac{m\,\omega^2}{2}\left[2\,\eta\,\Delta\eta - (\Delta\eta)^2\right] = m\,\omega^2\,\eta\,\Delta\eta\left(1 - \frac{\Delta\eta}{2\,\eta}\right) \qquad (10)$$

and the proportion of the energy lost in one cycle, known as damping capacity, is

$$\psi = \frac{\Delta U}{U} = \frac{m\,\omega^2\,\eta\,\Delta\eta}{m\,\omega^2\,\eta^2/2}\left(1 - \frac{\Delta\eta}{2\,\eta}\right) = \frac{2\,\Delta\eta}{\eta}\left(1 - \frac{\Delta\eta}{\eta}\right) \qquad (11)$$

Equation (11) also expresses the proportion of energy gained if exciting forces cause an increase of $\Delta\eta$ in the amplitude.

The damping forces can be conveniently classed as (a) structural (mechanical hysteresis, bearing and rivet friction, plastic yielding of deck and other elements, etc.) and (b) atmospheric (opposing inertial and viscous forces from still or moving air). It is the combined effect of all damping forces which causes any observed decay in the amplitude. In order to understand the damping action, therefore, it is necessary first to consider the character of viscous, hysteresis, friction and atmospheric damping.

(A) *Viscous Damping.*—If a damping force is directly proportional to the velocity of the vibrating system (as is true of viscous forces in a fluid) the curve representing the decay of amplitude will be the exponential curve (given in any book on vibration (4d)):

$$\eta = \eta_0 \, e^{-cgt/(2w)} \qquad \text{(12a)}$$

in which: η_0 is the initial amplitude; η is the amplitude at the end of a period of time, t; c is the damping force per unit velocity; g is the acceleration of gravity; and, w is the weight of the system which vibrates at the amplitude, η. Eq. 12a can also be expressed:

$$\eta = \eta_0 \, e^{-\delta N t} = \eta_0 \, e^{-\delta \, \omega t/(2\pi)} \qquad \text{(12b)}$$

in which $\delta = \dfrac{c\,g}{2\,w\,N} = \dfrac{\pi\,c}{m\,\omega}$, N being the frequency in cycles per second and $\omega = 2\,\pi\,N$ being the circular frequency in radians per second. Symbol δ is called the "logarithmic decrement" and for viscous damping, is constant for all amplitudes during a given vibration but is reduced if the frequency, ω, is increased without changing c or m. If $t = \dfrac{2\,\pi}{\omega}$, the length of one cycle, then $N\,t = 1$ and Eq. 12b expresses the ratio of two successive amplitudes:

$$\frac{\eta_1}{\eta_0} = e^{-\delta} \qquad \text{(13)}$$

Thus $-\delta = \log_e \dfrac{\eta_1}{\eta_0}$ or $\delta = \log_e \dfrac{\eta_0}{\eta_1}$.

It is difficult to measure accurately the change in amplitude in one cycle but the change in, say, ten cycles can be measured with satisfactory precision. If Eq. 12b is written for the time, $t = 10 \times \dfrac{2\,\pi}{\omega}$, it gives the ratio, $\dfrac{\eta_{10}}{\eta_0} = e^{-10\delta}$; and, $\delta = \dfrac{1}{10} \log_e \dfrac{\eta_0}{\eta_{10}}$. Thus the viscous damping force can be easily evaluated by measuring its effect on the amplitude.

The instantaneous position of a mass that is vibrating at a frequency, ω, and an amplitude, η, in simple harmonic motion may be expressed $\bar{\eta} = \eta \sin \omega\, t$. Its velocity $\dfrac{d\bar{\eta}}{dx} = \omega\,\eta \cos \omega\, t$, the maximum velocity being $\omega\,\eta$. A viscous damping force varying with the velocity may be expressed $F \cos \omega\, t = c\,\omega\,\eta \cos \omega\, t$ where c is the force at unit velocity. This force acts in phase with the velocity and in one cycle it performs work equal to $F\,\pi\,\eta = c\,\pi\,\omega\,\eta^2$ provided the damping is low so that the motion is essentially simple harmonic.

(While acting through the distance, $d\bar{\eta}$, the force performs work, $F \cos \omega\, t\, d\bar{\eta}$; $d\bar{\eta} = \frac{d\bar{\eta}}{dt}\, dt = \omega\, \eta \cos \omega\, t\, dt$; and, the work performed is $(F \cos \omega\, t)\, \omega\, \eta \cos \omega\, t\, dt$. In one cycle t ranges from 0 to $\frac{2\,\pi}{\omega}$; hence the work per cycle is $\Delta U = F\, \omega\, \eta \int_0^{2\pi/\omega} \cos^2 \omega\, t\, dt = F\, \omega\, \eta \int_0^{2\pi} \frac{1}{\omega} \cos^2 \omega\, t\, d(\omega\, t) = F\, \pi\, \eta$.) The relative decrease in energy is

$$\psi = \frac{c\, \pi\, \omega\, \eta^2}{m\, \omega^2\, \eta^2/2} = \frac{2\, \pi\, c}{m\, \omega} \qquad (14)$$

It will be noted that this energy ratio is just twice the value of δ as used in Eq. 12b. The same relationship can be derived by expressing

$$-\,\delta = \log \frac{\eta}{\eta_0}$$

—by the first two terms of the series for natural logarithm:

$$\frac{\eta_1}{\eta_0} = \frac{\eta_0 - \Delta\eta}{\eta_0} = 1 - \frac{\Delta\eta}{\eta_0}$$

$$-\,\delta = \log_e \left(1 - \frac{\Delta\eta}{\eta_0}\right) = -\,\frac{\Delta\eta}{\eta_0} - \frac{1}{2}\left(\frac{\Delta\eta}{\eta_0}\right)^2 = -\,\frac{\Delta\eta}{\eta_0}\left(1 + \frac{\Delta\eta}{2\,\eta_0}\right);$$

and,

$$\delta = \frac{\Delta\eta}{\eta_0}\left(1 + \frac{\Delta\eta}{2\,\eta_0}\right) \qquad (15)$$

Comparing Eq. 15 with Eq. 11 and noting that both η and η_0 as used represent the higher of two successive amplitudes it is seen that, very nearly,

$$\psi = 2\,\delta\, \frac{1 - \delta/2}{1 + \delta/2} \qquad (16)$$

If δ is small, say less than 0.05, the error in using $\psi = 2\,\delta$ is not serious but for larger values of δ the damped vibration differs from simple harmonic motion enough to require the corrective term. If δ exceeds 0.40 a still more refined correction is needed.

(B) *Hysteresis Damping.*—Hysteresis damping arises from the absorption of energy due to the imperfectly elastic behavior of the stressed element (1i). The stress-strain curve is not perfectly straight and is slightly different for descending than for ascending loads as shown by Fig. 9 which may represent the behavior of a simple spring, such as a wire supporting a weight.

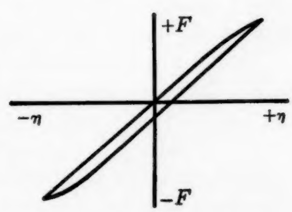

Fig. 9.—Hysteresis Loop

The area enclosed within the stress-strain curves for a complete, closed cycle represents force times distance and indicates the energy lost per cycle. Both the stress and the strain vary directly with the amplitude of the displacement. Therefore, the energy loss is some constant times η^2. For this reason authorities usually consider the energy loss arising from hysteresis as the work done by a

damping force which is proportional to the amplitude and in phase with velocity (that is, equal to $C\,\eta \cos \omega t$) and the work it performs in a cycle is $C\,\pi\,\eta^2$ C being a constant, the damping force per unit amplitude, to be determined experimentally. The damping capacity then is

$$\psi = \frac{C\,\pi\,\eta^2}{m\,\omega^2\,\eta^2/2} = \frac{2\,\pi\,C}{m\,\omega^2} \qquad (17)$$

It would appear by comparing Eqs. 14 and 17 that hysteresis damping is even more sensitive to the frequency, ω, than is viscous damping but this is not the case because the hysteresis damping represented by C in the numerator all arises from the action of the spring which also determines the value of ω^2 in the denominator. This can be shown as follows:

If k is the spring constant and η the displacement, then the force on the spring is $k\,\eta$ and the stress may be expressed as $C_1\,k\,\eta$, C_1 being the constant of proportionality relating the spring force and the stress in the particular spring. The damping force, proportional to the stress, is $C_1\,C_2\,k\,\eta$, C_2 being a constant of proportionality. The damping force per unit amplitude, then, is $C_1\,C_2\,k$ which can be substituted for C in Eq. 17. Then

$$\psi = \frac{2\,\pi\,C_1\,C_2\,k}{m\,\omega^2} \qquad (18)$$

The natural frequence of a spring is $\omega = \sqrt{\frac{k}{m}}$; hence $k = m\,\omega^2$. If the frequency is changed without affecting m then, automatically, k must change in the same ratio as ω^2 and ψ is unchanged; ω^2 can also be altered without affecting k by changing m in the inverse ratio but the product, $m\,\omega^2$, will then be unchanged and ψ remains constant. In fact, k cancels $m\,\omega^2$ and $\psi = 2\,\pi\,C_1\,C_2$ —that is, it depends only on the hysteresis curve of the material (which determines C_2) and the design of the spring (which determines C_1). Carried still further, it is independent of the spring design because both the damping force (which absorbs energy) and the elastic force (which stores energy) are proportional to C_1 which cancels out in expressing the ratio of the absorbed to stored energy. In the last analysis the hysteresis damping capacity is a function of the hysteresis curve only.

While Eq. 17 indicates ψ to be independent of the amplitude, η, tests show that it increases slowly with amplitude and stress, which means that the damping force actually increases as some fractional power of η a little greater than 1.

(C) *Coulomb Friction Damping.*—The friction force is constant and does not depend upon the amplitude or frequency and the work it does in a cycle may be expressed as some constant, C_3, times η. The damping capacity, proportion of energy of vibration damped out per cycle, then is

$$\psi = \frac{C_3\,\eta}{m\,\omega^2\,\eta^2/2} = \frac{2\,C_3}{m\,\omega^2\,\eta} \qquad (19)$$

For such damping the logarithmic decrement decreases as the amplitude increases. The curve of the decay of amplitude, plotted against time is a straight line rather than the exponential curve in Eq. 12b.

Notwithstanding the general statement above, Mr. Bleich has shown that under certain conditions the movement of a frictional force may be proportional to the square of the movement of the oscillating mass so that its work varies as η^2 and it then has the character of hysteresis damping. However, this second type of friction damping is small (1k).

(D) *Structural Damping.*—The character of the structural damping of a complicated structure depends upon the relative proportions and character of its hysteresis and friction components. The damping capacity may increase or decrease with amplitude or be nearly constant. Some field tests on light suspension bridges have shown it increasing up to a certain amplitude and then decreasing (3e). This form of curve may be attributed to the gradual increase in sources of friction as the increasing stress (due to increasing amplitude) overcomes the static friction and causes motion to begin progressively at various contacting surfaces. This increase in sources of friction causes a progressive increase in the overall frictional force. When the amplitude increases further after all sources of friction are working, the damping capacity decreases and the curve approaches the hyperbolic form characteristic of simple friction damping. The possibility of this type of damping was anticipated by Mr. Bleich in his theoretical studies (1l).

(E) *Atmospheric Damping.*—The atmospheric damping forces arise from the relative movement of the air and the vibrating body and may be viscous or inertial. In the case of bridge structures the flow is essentially turbulent and the forces are mostly inertial—that is, the force varies as the square of the velocity as stated by Eq. 1. In still air the velocity is entirely due to the vibration and is equal to $\omega\,\eta \cos \omega\, t$. The force is then proportional to $\omega^2\,\eta^2$ and its work is proportional to $\omega^2\,\eta^3$ and the damping capacity may be stated,

$$\psi = \frac{C_4\,\omega^2\,\eta^3}{m\,\omega^2\,\eta^2/2} = \frac{2\,C_4\,\eta}{m} \qquad (20)$$

which is independent of frequency and increases with amplitude. In Eq. 20, C_4 is an experimental constant.

In a wind stream the relative velocity is determined by both the wind and the motion of the body. The resulting forces and the work they perform in a cycle do not bear the simple relation to ω and η such as represented by Eqs. 14, 17, 19 and 20. In fact, at certain velocities their sign or phase may change with respect to the motion and they may develop resonance and tend to increase the amplitude—that is, they may excite motion. Under these circumstances their energy is a contribution to, rather than a dissipation of, the energy of vibration. Mathematically their effect can be treated simply as a negative damping whose effect on the vibration can be measured by a negative logarithmic decrement.

APPENDIX III.—BEHAVIOR OF EXISTING BRIDGES

1. Bronx–Whitestone

The data concerning movement of the bridge as indicated in Table I are taken from an unpublished "Report to the Triborough Bridge Authority on

Behavior of the Bronx-Whitestone Bridge under Wind Action and on Provisions Made to Stiffen the Bridge" by O. H. Ammann, dated July 23, 1945. This report includes a chart of daily measurements of bridge movement and of maximum wind velocities measured either at the bridge site or in lower Manhattan and describes the center cable ties, end frictional devices and diagonal tower stays which were installed to restrain the oscillation. Center ties and frictional devices were installed early, proved fairly effective and were progressively increased in size and effectiveness. Of stay ropes the report states:

> "While the stay ropes have had a considerable stiffening effect insofar as the ordinary moderate oscillations are concerned and in this respect confirm the conclusions derived from the model tests (dynamic tests in still air at Princeton University) it is evident that their effect has not been sufficient to prevent nor apparently reduce the exceptional oscillations of larger amplitude."

This observation is in agreement with the results of wind tunnel tests at the University of Washington (see Art. 3H).

The report indicates that the severest motions occurred most frequently during northwest winds (approximately 30° to 45° from the axis of the bridge) and that strong upward wind currents have been noticed outside of and through openings in the bridge floor. It states the early conclusion "that the motions were principally the result of longitudinal wind pressure acting from below on the pockets formed by the floor structure," and suggests also the explanation that the prevailing strong gusty winds come from the northwest. The severe oscillations occurred on days when the wind velocity was low or moderate but not high. The report was prepared prior to the widening of the roadway and the addition of the truss. Motion is reported to be negligible since this work was done.

2. Fykesund, Norway

The data concerning this bridge are taken from plans furnished by Mr. Selberg, and his letter of January 24, 1947, to Mr. Farquharson from which the following is quoted:

> "The Fykesund bridge has shown vertical oscillations with 1, 2, or 3 nodes and amplitudes about $\pm 0.8\ m$. Positive reports of torsional oscillations are never received.
>
> "The Fykesund bridge lies in a very stormy place, and the wind direction is frequently varying, both vertically and horizontally. The oscillations have mainly occurred for wind directions in or out the fiord.
>
> "The wind velocity has never been measured; we had no occasion for it during the war. Since the erection of stays in 1945 the bridge has shown no oscillations, however, we have arranged for measuring eventual oscillations, and wind velocities."

The map shows the bridge about one and a half miles from the mouth of Fyksesund with a fetch of about five miles across the main fiord for a normal wind from the southeast. The stays mentioned are attached to the deck at points approximately 100 ft and 145 ft from each tower and extend downward to the piers at angles of about 60° and 68° respectively to the vertical.

3. Beauharnois, Quebec, Canada

Data concerning this bridge and its movements have been given by D. B. Armstrong, of the Dominion Bridge Company, Limited, in correspondence with G. S. Vincent, M. ASCE.

Inasmuch as this bridge represents a new addition to the record of modern bridge movements it is desirable to present here data concerning its characteristics as tabulated below:

Span, in feet	580
Sag, in feet	58
Horizontal length of unloaded backstays, in feet	178.75
Distance between centers of cables and girders, in feet	30.0
Roadway width, in feet	28
Sidewalk width (one walk along outside north girder), in feet	4
Weight, in pounds per foot per cable—	
Dead load	1715
Cable only	100
Depth of plate girders, back to back of angles, in feet	7.5
Average moment of inertia per girder (including a moderate allowance for the stiffness of the deck), in in.^{2}–ft^2	1000
Net area of one cable (thirty-seven $1\frac{1}{8}$-in. strands), in square inches	27.71
Top of roadway 3.5 ft below top of girder	...
Deck, 3-in. I-beam Lok filled with concrete	...
Curbs, 9-in. concrete (sealed to girder web by $\frac{1}{4}$-in. plate)	...
Stringers—	
Roadway, seven lines 16-in. WF beams at 45 lb	...
Sidewalk, one 10-in. 15.3-lb channel	...

The center cable bands are connected to the stiffening girders by means of rigid diagonal struts, designed to prevent relative longitudinal motion between the cable and girders. The laterals are located in the plane of the bottom flanges of the floor beams and the floorgrid is, in turn, welded to the tops of the stringers and floor beams. This constitutes a closed section having the depth of the floor beam and providing effective torsional resistance.

Mr. Armstrong furnished a copy of a report on observations made by engineers of Beauharnois Light, Heat and Power, Consolidated and a print showing temporary stays installed in 1950 extending from the tower tops to points on the span 40 ft and 80 ft from the towers. He also stated:

> "The bridge runs East and West, close to the shore of Lake St. Louis which provides an uninterrupted sweep of 12 miles for wind from the Northeast. On the South side the power house, 120 feet high above lake level and over 2000 ft long, parallels the bridge at a distance of about 900 feet and shields it fairly well from critical South westerly winds.
>
> "When being designed the stiffness of the bridge was checked by both the Ammann and Steinman formulas and appeared to have twice the stiffness required. However, under a steady moderate diagonal wind it rises and falls with a constant frequency of 33 cycles per minute, with both cables in phase and no sign of a longitudinal wave action.
>
> "It has been suggested that the topography of the site, the unbalanced sidewalk to the North side and the elasticity of the single strand hangers may all have something to do with it.
>
> "Since the diagonal stays were installed, there have been no winds of sufficient velocity to test their effectiveness but a close watch is being kept

on the bridge by the engineers of Beauharnois, Light, Heat & Power Company. It may be of interest to know that previously, without stays, the bridge remained motionless under normal steady winds of up to 50 mph and that oscillations only occurred under diagonal wind."

The tabulation of observed motions of the bridge indicates that only the first symmetric vertical mode has been observed (see Fig. 10a). Its observed

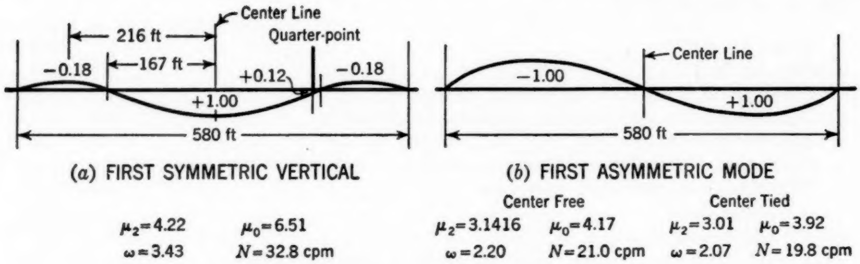

FIG. 10.—COMPUTED WAVE FORMS

frequency was thirty-three cycles per minute; the computed frequency is 32.8. The computed wave form indicates that the amplitude at the quarter point is about one-eighth of that at the center line. This probably accounts for the fact that there is only one observation of an amplitude at the quarter point and that is only $\frac{1}{4}$ in.

The asymmetric mode appears to be inhibited by the center cable ties and the added damping which they cause by forcing a longitudinal sway of the suspended structure with accompanying end friction.

4. GOLDEN GATE

Data concerning the movements of the Golden Gate Bridge have been taken from the records of the cooperative research project of the Golden Gate Bridge and Highway District and the Bureau of Public Roads. A complete report on this project is to be published.

5. LIONS GATE

The following is quoted from a letter of June 3, 1943, addressed to Mr. Ammann by P. L. Pratley, M. ASCE:

"On April 23rd a very severe gale of short duration was experienced in the Burrard Inlet across which the Lion's Gate Bridge is built. This gale is supposed to have been the heaviest in the neighborhood for some 15 years and is therefore the most severe the bridge has experienced. I was in Vancouver on inspection of the structure at the time and am therefore well acquainted with the situation. The wind blew very strongly, and in heavy gusts from the east at about 7 AM and reversed itself entirely to become a west wind of about equal intensity by about 7:30 AM and persisted in this direction and at about this strength for about 1 hour. The direction was maintained but the force was reduced for the next three hours and then again just before noon, a very heavy 'blow,' lasting for some 15 or 20 minutes, and almost equal in intensity with the heaviest wind experienced

in the early morning, completed the gale. I have on official figures of the wind velocity but would imagine it was between 50 and 60 miles an hour as I saw people literally blown off their feet on the main east and west streets of Vancouver and from information given me by the naval personnel no one could possibly walk out on the structure except by holding on to the fence which these officials were compelled to do. The gale was apparently quite unexpected and being of such short duration we had no opportunity of setting up instruments for scientific measurement.

"In the early morning I am assured that some vertical vibration was perceptible from the Signal bridge which is an observation and control structure built across the roadway at the center of the main span from which the marine and naval authorities command a perfect view up and down the narrows. The officer in charge told me that he could detect vertical movement by sighting one or more of the suspender sockets against the railway structures on the north shore. The sockets in question would be 300 or 400 feet from the Signal Bridge and the railway span referred to is about 1800 ft from the same Signal Bridge. His impression was that the structure was vibrating slightly with a maximum amplitude of about 3 inches, this occurring at the center of the main span so that the 1550 ft would be oscillating without nodes.

"I further learned that one of the maintenance staff was below the structure at the time and he also felt quite sure that he could detect a slight vertical vibration which appeared to arise with the heavy gusts and then to quickly dampen down. He ventured no figures as to the amplitude but agreed that the 3″ mentioned by the signal officer would be a perfectly safe maximum figure. He also suggested that this vibration arose 3 or 4 times during the gale and fell away to nothing as each heavy gust subsided.

"Both officials stated definitely that there was no indication of lateral movement and this is substantiated by my own observations just before noon when the final heavy gust occurred. I was then on the structure and from the South Main Tower I sighted, with the naked eye, the Signal Bridge cabins against the toll houses at the north end, the intercept being 775 feet from me to the Signal Bridge and about 4200 feet from the Signal Bridge to the toll house. For a period of several minutes I could detect no vibration and no restitutional movement.

"As to frequency, I of course have no reliable figures but the signal officer illustrated to me the approximate frequency by the movement of his hands and these indications would suggest something of the order of 12 or 15 cycles per minute. Also, at noon I could detect no vertical vibration whatsoever either by standing on the approach spans or by standing on the suspended span. At the same time I feel that there must have been some very minute vertical movement, and that its repercussions finally reached the bearings at the extreme south end of the south side suspended span because during my examination of these bearings before noon 'blow' had completely ceased I thought I could detect very small vertical movements of the steel sliding plates in the bronze guides as the grease seemed to be alternately sucked in the exuded. Such movements would be in the nature of a few thousandths of an inch only and in half an hour's time they had disappeared.

"I also immediately inspected the main sliding and rotating devices on the towers and found perfectly normal movements and nothing beyond normal movements had taken place at the shear bearings where wind reaction is transmitted to the tower. Furthermore, I examined the north backstay cables which, although unsupported for over 400 feet, had certainly not swayed sufficiently in any plane to touch the viaduct steel which they normally clear by about 1 inch.

"Where these cables depart from the saddles on the North Cable Bents nothing more than the usual hair crack in the red lead paste, which acts as caulking from cable to saddle seat, could be detected. Up on the top of the towers, over the main saddles, there was absolutely no indication of such hair cracks having been caused and as those points have been consistently examined year after year I regard this as very sound evidence that the structure has sufficient rigidity in lateral as well as vertical planes.

"On the whole, I am thus very well satisfied that the span stood up to this heavy wind condition without suffering in any particular and without experiencing any noticeable or abnormal movements objectionable or unobjectionable."

6. Peace River

The following is quoted from a letter from Raymond Archibald, M. ASCE, who was in charge of the design and construction of this bridge and describes his observations during opening ceremonies in July, 1943:

> "The Peace River Bridge was dedicated and opened to traffic in July 1943. I was seated in the stand located at the center of the main span and a strong steady wind was blowing from an upstream direction normal to the bridge. It is my estimate that this wind reached a velocity of 40 or 50 miles per hour at times and held this high velocity for periods from 3 to 5 minutes in length. When the wind was steady and of high velocity, the bridge definitely moved. My observations were made by sighting the positions of the truss with respect to the horizon. This movement reached proportions which gave me a slight sensation of sea-sickness. I believe it was primarily a torsional motion."

Targets and an anemometer were placed on this bridge and a transit was set up to observe its motion in the wind over a period of two weeks in July, 1947. During this period the wind velocity generally ranged up to about twenty-five miles an hour, rarely reaching the latter figure, and the observed double amplitude never exceeded one inch. These smaller motions appeared to be essentially vertical and had a period of about 3.5 seconds. (The computed period for the first symmetric vertical mode is 3.8 seconds; the observed period being shorter than this suggests that the motion was coupled, with the frequency shifted somewhat toward that of the torsional mode, that is toward a shorter period.)

Joe Trca, living near the Peace River Bridge, has described his observations during a windstorm late in September, 1947, during which velocities of 70 mi per hr were reported by radio. He estimated the maximum at 50 mi per hr during his observations. He found difficulty in walking due to the motion of the deck and discomfort and uneasiness when he sat on the curb to note the torsional motion.

7. Liard River

Transit and anemometer readings were made on this bridge for two weeks in August, 1947, during which time the wind reached a velocity of 35 mi per hr. The maximum double amplitude (2 in.) was observed in a wind of 20 mi per hr to 25 mi per hr. It was specifically noted that the higher winds were turbulent and gusty and motion only built up after the gust died down to a steady wind of

about 20 mi per hr. Frequently it was observed by noting the movements of the trees that a high wind was limited to a small area. Such winds had little effect upon the bridge.

This bridge is subject to much greater movements at other seasons when higher winds are more frequent. The following is quoted from the report (unpublished) on the observations made in 1947:

> "Greater wind and motion have been observed by others at Liard River Bridge also. Again, no attempt has been made to give the amplitude, but Lieutenant Baldock (Bridge Engineer, Northwest Highway System) reported he had been on this bridge when the motion of individual longitudinal planking of the deck could be observed. This could not have happened except with much greater amplitude than noted during the present series of observations, and it appears quite likely that a considerable torsional motion must have been involved."

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